f-Continuous Operators on $B(L^2M)$ and the Families Index

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Abstract

In this work, we present and explore a new class of operators in $B(L^2M)$, defined for any smooth, compact manifold M, and their connection to K-theory. These operators form a C^*-algebra, $C^*(M)$, consisting of what we define as f-continuous operators. Notable examples of these operators are the compact operators and the pseudodifferential operators of order zero. These operators also allow us to define norm continuous families of operators associated to a fiber bundle $p : Z \to Y$ with compact fiber. We will then show that norm continuous families of f-continuous operators associated to this bundle form another C^*-algebra, $C^*(Z, Y)$. We will then define a families index for elements of $C^*(Z, Y)$, and show that this matches the definition given by Atiyah-Singer. Finally, we will show that the boundary map in analytic K-theory gives the families index, cementing the Atiyah-Singer construction in this context.