Practice Exam 2

Instructions. Show all your work. Full credit may not be given for correct answers if they are not adequately justified. The exam is closed book: no notes or calculators are permitted.

Good luck!

1. Find parametric equations for the line segment from $(2, 1, 4)$ to $(5, 2, 1)$. Be sure to indicate the domain of your parameter.

2. Find the equation of the plane containing the point $(1, 2, 3)$ and the line

$$\frac{x-2}{5} = \frac{y-4}{2} = \frac{z-4}{3}$$

3. Evaluate the following integrals:

   (a) $\int \frac{\ln(x)}{x^2}\,dx$

(b) $\displaystyle\int \frac{x\ln(1+x^2)}{1+x^2}\,dx$

(c) $\int \tan^4(x)\,dx$

(d) Evaluate $\displaystyle\int \frac{x+1}{\sqrt{x^2+2x+2}}\,dx$

4. A woman exerts a horizontal force of 25 pounds as she pushes a box up a ramp that is 10 feet long and inclined at an angle of 30 degrees above the horizontal. Find the work done on the box.

5. The line

$$l_1: \; x = t, \, y = 2t + 1, \, z = t + 4$$

intersects one of the following two parallel lines. Determine which of the lines it intersects and find the intersection point.

$$l_2: \; x = 1 + 2t, \, y = 2 + 5t, \, z = 6 + t$$

$$l_3: \; x = 1 + 2t, \, y = 2 + 5t, \, z = 5 + t$$

6. Consider the curve defined by the vector-valued function

$$\mathbf{r}(t) = \langle t, t^2, t^3 \rangle.$$

(a) Find the tangent line to this curve at the point with parameter $t = 1$.

(b) (Recall we are considering the curve $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$.) Find all points on this curve at which the tangent vector to the curve is parallel to the plane $x - 2y + z = 0$. (You can specify the points either by giving their coordinates or by just specifying the parameter value t.)

7. Sketch several level curves of the function $f(x, y) = \frac{x^2+y^2}{x}$

8. Multiple choice. Circle the correct response. No partial credit will be given.

   (a) Let $\mathbf{u}$ and $\mathbf{v}$ be non-parallel vectors and denote the scalar projection of $\mathbf{v}$ onto $\mathbf{u}$ by $\text{comp}_{\mathbf{u}}\mathbf{v}$. If $\text{comp}_{\mathbf{u}}\mathbf{v} = -2$, then the angle between $\mathbf{u}$ and $\mathbf{v}$ is

   $A.\ < \frac{\pi}{2}$ $B.\ \frac{\pi}{2}$ $C.\ > \frac{\pi}{2}$ $D.\ \pi$ $E.$ None of these

   (b) The parallelepiped spanned by the vectors $\langle 1, 0, 2\rangle$, $\langle 3, 1, 1\rangle$ and $\langle 1, 2, 5\rangle$ has volume

   $A.$ 8 $B.$ 9 $C.$ 10 $D.$ 13 $E.$ None of these

   (c) If $\mathbf{v} \cdot \mathbf{w} = 0$, then $\mathbf{v} \times (\mathbf{v} \times \mathbf{w})$ is

   $A.$ Perpendicular to $\mathbf{w}$ $B.$ Equal to the zero vector $C.$ Parallel to $\mathbf{w}$ $D.$ Not defined $E.$ None of these

   (d) A particle moving in space has acceleration at time t given by

   $$\mathbf{a}(t) = \langle 2, 6t, 12t^2\rangle$$

   and has initial velocity $\mathbf{v}(0) = \langle 1, 0, 0\rangle$. Then its velocity $\mathbf{v}(t)$ at time t is

   $A.\ \langle 3, 3, 4\rangle$ $B.\ \langle 0, 6, 24t\rangle$ $C.\ \langle 2t + 1, 3t^2, 4t^3\rangle$ $D.\ \langle 2t + 1 + C_1, 3t^2 + C_2,$
   $4t^3 + C_3\rangle$ $E.$ None of these