MATH 8: Practice Exam I

NOTE. The goal of this practice exam is to allow you to see some possible formats of exam problems (a blend of long answer and multiple choice) and to get some practice. We do not claim that this practice exam is comparable to the midterm exam with regard to difficulty, length and distribution of subject matter. In particular, no exam can cover every topic. The fact that a particular concept may or may not appear in this practice exam is **not** an indicator of whether it will appear on the actual exam.

1. Find the limits of the following sequences or show that they diverge:

   (a) $\{\frac{\ln(n)}{\ln(2n)}\}$

   (b) $\{\frac{1\cdot 3\cdot 5\cdots(2n-1)}{n!}\}$

2. Determine whether the following series converge or diverge. Indicate which method you use and verify that the series satisfies the conditions needed for the chosen method.

   (a.) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n+1}{n^2}$

   (b.) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n^2+\sin(n)}{3n^2+1}.$

   (c) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n!}{5^n(2n)!}$

3. Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n(x-2)^n}{\sqrt{n}7^n}$
.

4. Using the minimum number of terms necessary, approximate $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^n}$ to within 0.01. (You can leave your answer as a sum.) Is your estimate an overestimate or an underestimate of the actual value?

5. (a) By beginning with the geometric series, find the Maclaurin series for the function $\frac{3x^2}{(1-2x)^2}$. Give the entire series.

   (b) What is the radius of convergence of the series in part (a)?

6. Let $f(x) = \ln(x)$. Find the Taylor series for f centered at $a = 1$.

7. (a) What is the smallest positive integer n for which the limit

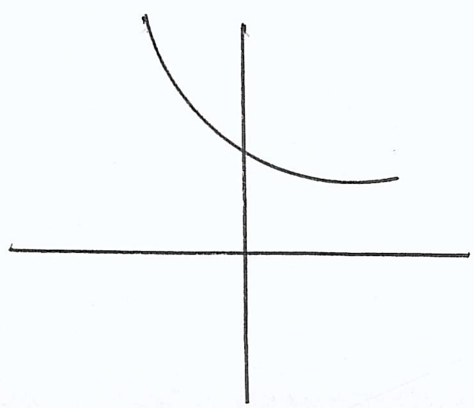
$$\lim_{x \to 0} \frac{e^{x^2} - 1 - x^2}{x^n}$$

exists and is nonzero?

(b) For the number n you found above, what is the value of this limit?

8. The graph of a function $f(x)$ is shown below. The questions below refer to the Maclaurin series for $f(x)$ given by $\sum_{n=1}^{\infty} c_n x^n$.

1. Is c_0 positive or negative? (Explain your answer.)
2. Is c_1 positive or negative? (Explain your answer.)
3. Is c_2 positive or negative? (Explain your answer.)



Multiple choice. For the following problems, circle the correct answer.

9. Suppose $\sum_{n=1}^{\infty} a_n$ converges, and for each n, $a_n \neq 0$. Which of the following statements must be true?

A. $\sum_{n=1}^{\infty} \frac{1}{a_n}$ converges.

B. $\sum_{n=1}^{\infty} \frac{1}{a_n}$ diverges.

C. $\sum_{n=1}^{\infty} |a_n|$ converges.

D. None of the above.

10. Suppose that you are able to use the ratio test to show that $\sum_{n=1}^{\infty} a_n$ converges. Which of the following statements is correct?

A. $\sum_{n=1}^{\infty} |a_n|$ converges.

B. $\sum_{n=1}^{\infty} a_n^2$ converges.

C. $\sum_{n=1}^{\infty} na_n$ converges.

D. All of the above are true.

E. None of the above are true.