

Final Exam

Math 3

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Name: _____

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Instructions: The Final Exam is multiple choice. You are not allowed to use calculators, books, or notes of any kind. All your answers to the questions must be marked on the Scantron form provided. Take a moment now to **print your name and section clearly on your Scantron form**, and on your exam booklet. You may write on the exam, but you will only receive credit for what you write on the Scantron form. At the end of the exam you must turn in both your Scantron form, and your exam booklet. There are 30 multiple choice problems each worth 5 points, for a total of 150 points. Check to see that you have 15 pages of questions. Good luck. We have enjoyed working with you this term and hope to see you again in the future.

MULTIPLE-CHOICE

1. The limit $\lim_{x \rightarrow \infty} \frac{2x^3 + \sqrt{x^2 + 1}}{x^3 - 4}$ equals:

- (a) ∞
- (b) 0
- (c) 1
- (d) 2
- (e) Does not exist.

2. The limit $\lim_{x \rightarrow e} \frac{\ln x + 1}{\ln x - 1}$ equals:

- (a) 1
- (b) e
- (c) 0
- (d) ∞
- (e) Does not exist.

3. The limit $\lim_{h \rightarrow 0} \frac{\sin(\frac{\pi}{2} + h) - 1}{h}$ equals:

(a) 1

(b) 0

(c) -1

(d) $\frac{\sin h}{h}$

(e) Does not exist.

4. At the point $x = -1$, the function $f(x) = |x + 1| + 2$ is:

(a) Continuous and differentiable.

(b) Continuous but not differentiable.

(c) Differentiable but not continuous.

(d) Neither continuous nor differentiable.

(e) Differentiable and the derivative is 0.

5. For the function $f(x) = \frac{x^2 - 9}{x - 3}$, the point $x = 3$ is:

- (a) An inflection point.
- (b) A local maximum.
- (c) A local minimum.
- (d) A removable discontinuity.
- (e) None of the above.

6. The slope of the tangent line to the graph of $f(x) = \frac{2^x}{e^x + 1}$ at the point $(0, 1/2)$ is:

- (a) $(2 \ln 2 - 1)/4$
- (b) $1/2$
- (c) $(2 \ln 2 - 1)/2$
- (d) $(2 \ln 2 + 1)/2$
- (e) None of the above.

7. The length of the arc of the curve $y = \frac{x^2}{2} - \frac{\ln x}{4}$ from $(1, \frac{1}{2})$ to $(e, \frac{2e^2-1}{4})$ is:

(a) $\frac{e^2}{2} - \frac{1}{4}$

(b) $e + 1$

(c) $\frac{1}{2e}$

(d) $e + \frac{1}{4}$

(e) None of the above.

8. The x -coordinate of the centroid of the region below the curve $y = x - x^2$ and above the x -axis is: [Note to Math3 Fall07 students: You will not be responsible for a problem like this, but you will need to know how to find the centroid of, say, a collection of rectangles.]

(a) $1/12$

(b) $1/6$

(c) $1/2$

(d) 0

(e) 1

9. The integral $\int_0^{\pi/4} \cos x (\sin x)^2 dx$ is:

(a) $\frac{\pi^3}{192}$

(b) $\frac{1}{3\sqrt{8}}$

(c) $\frac{\pi}{2}$

(d) 0

(e) None of the above.

10. The integral $\int_0^{3/4} \frac{1}{\sqrt{9 - (2x)^2}} dx$ is:

(a) $\pi/12$

(b) $\pi/6$

(c) $\pi/2$

(d) $\pi/3$

(e) $\pi/4$

11. The area of the region bounded by the curves $y = x^4 + x^2 + 2$ and $y = -x^3 + x^2 + 2$ is

(a) $-1/20$

(b) $1/20$

(c) $9/20$

(d) $-9/20$

(e) 0

12. The value of $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} e^{i/n}$ is

(a) $\frac{1}{n} e^{1/n}$

(b) $\frac{1}{e}$

(c) $e - 1$

(d) e^x

(e) ∞

13. If $F(x) = \int_{x^2}^1 e^{\sqrt{t}} dt$, then $F'(0)$ is

- (a) $2e - 1$
- (b) $-e$
- (c) 1
- (d) 0
- (e) None of the above.

14. The Trapezoidal rule approximation T_3 for $\int_0^1 \frac{1}{x^2 + 1} dx$ is equal to

- (a) $\frac{1}{6} \left(1 + \frac{1}{2} + 2 \left(\frac{9}{10} + \frac{9}{13} \right) \right)$
- (b) $\frac{1}{3} \left(\frac{3}{2} + 2 \left(\frac{9}{10} + \frac{9}{13} \right) \right)$
- (c) $\frac{1}{6} \left(1 + \frac{1}{2} + \frac{9}{10} + \frac{9}{13} \right)$
- (d) $\frac{1}{3} \left(\frac{1}{2} + \frac{9}{10} + \frac{9}{13} \right)$
- (e) None of the above.

15. The integral $\int_0^1 \frac{1}{3x+1} dx$ is:

- (a) $\frac{1}{3} \ln 4$
- (b) $\frac{1}{3}(\ln 4 - 1)$
- (c) $\ln 4$
- (d) $3 \ln 4$
- (e) $3(\ln 4 - 1)$

16. The antiderivative of $f(x) = x \ln x$ is

- (a) $x \ln x + C$
- (b) $\frac{1}{2} \ln x^2 + C$
- (c) $\frac{1}{2} x^2 \ln x + C$
- (d) $\frac{1}{2} x^2 \ln x - \frac{x^2}{4} + C$
- (e) $\frac{1}{2}(x^2 \ln x - \ln x^2) + C$

17. The slope of the line tangent to the curve $y \arctan x + y^2 = 1$ at the point $(0, 1)$ is

- (a) $1/2$
- (b) $-1/2$
- (c) $\pi/2$
- (d) 0
- (e) Does not exist.

18. The linearization of the function $f(x) = 2 - \int_{e^e}^x \ln(\ln t) dt$ at $x = e^e$ is

- (a) $\ln(\ln x)$
- (b) $2 - x + e^e$
- (c) x
- (d) $2 - x$
- (e) $\ln(\ln x) - 1$

19. Let

$$f(x) = \begin{cases} kx + 1 & x \geq 0, \\ x^3 - 2x + 1 & x < 0, \end{cases}$$

where k is a constant. For what value of k is f differentiable at $x = 0$?

- (a) For any value of k .
- (b) Only for $k = 1$.
- (c) Only for $k = -2$.
- (d) Only for $k = 0$.
- (e) No value of k makes f differentiable at $x = 0$.

20. If $f(x) = \sqrt{\cos(\ln x)}$, then $f'(e^{\pi/4})$ equals

- (a) $-\frac{1}{2^{5/4}e^{\pi/4}}$
- (b) $-\frac{1}{2^{5/4}}$
- (c) $\frac{1}{2^{3/4}}$
- (d) $\frac{1}{2^{1/4}e^{\pi/4}}$
- (e) None of the above.

21. Suppose that we apply Newton's method to approximate the root of the equation $f(x) = 3^x - x^3$. If we start at $x_0 = 2$, then, after one iteration of the method, x_1 is:

(a) 2

(b) $\frac{1}{9 \ln 3 - 12}$

(c) $2 + \frac{1}{3}$

(d) $2 - \frac{1}{9 \ln 3 - 12}$

(e) None of the above.

22. The solution to the initial value problem $y' = xe^x$, $y(0) = 0$ is

(a) $y = e^x + xe^x - 1$

(b) $y = \frac{x^2 e^x}{2}$

(c) $y = xe^x - e^x + 1$

(d) $y = xe^x - e^x$

(e) None of the above.

23. Consider the initial value problem $y' = x^2y$, $y(0) = 2$. Using Euler's method to find an approximation of the solution curve that starts at $(0, 2)$ using steps of size $1/2$, what is the approximate value of $y(1)$?

- (a) 1
- (b) $9/4$
- (c) 2
- (d) $5/2$
- (e) None of the above.

24. The solution to the initial value problem $y' = xy + x$, $y(0) = 2$ is

- (a) $y = 3e^{x^2/2} - 1$
- (b) $y = e^{x^2/2} - 1$
- (c) $y = \ln x - 1$
- (d) $y = \frac{x^2}{2} + 2$
- (e) None of the above.

25. The slope of the line tangent to the graph of the solution of the initial value problem $\frac{dy}{dx} = x^2y^2 + e^xy$ at $(0, 1)$ is

- (a) 1
- (b) 0
- (c) -1
- (d) e
- (e) None of the above.

26. Suppose that the number of rabbits in a certain farm follows an exponential model (that is, the growth rate of the population is proportional to its size). Assuming that there were 100 rabbits one year ago, and that there are 200 today, how many rabbits are there going to be a year from today?

- (a) 300
- (b) 400
- (c) 500
- (d) 600
- (e) 800

27. Suppose an object moves on the x -axis so that its position at time t is given by $x(t) = 2t^3 - 3t^2 + 1$. Then the object is at rest

- (a) When $t = -1/2$ and $t = 1$.
- (b) When $t = 0$ and $t = 1$.
- (c) Only when $t = 1$.
- (d) Never.
- (e) None of the above.

28. The object from problem 27 is speeding up on

- (a) $(1, \infty)$
- (b) $(1/2, 1)$
- (c) $(-\infty, 0) \cup (1, \infty)$
- (d) $(0, 1/2) \cup (1, \infty)$
- (e) None of the above.

29. Let $f(x) = 3x^4 + 8x^3 + 6x^2 - 1$. Then $f(x)$ is increasing on

- (a) $(-1, \infty)$
- (b) $(-1, 0)$
- (c) $(-\infty, -1) \cup (0, \infty)$
- (d) $(0, \infty)$
- (e) None of the above.

30. How many inflection points does $f(x) = 3x^4 + 8x^3 + 6x^2 - 1$ have?

- (a) 0
- (b) 1
- (c) 2
- (d) 3
- (e) More than 3.