

# Math 24 Spring 2006 Quiz 1

## Review Guide

**A.** To look at specifically (besides the homework, lecture notes, and book sections in general):

- (1) The true-false questions at the end of each section 1.2–1.6.
- (2) The key examples:  $F^n$ ,  $P_n(F)$  and  $P(F)$ ,  $M_{n \times m}(F)$ ,  $\mathcal{F}(S, F)$ .
- (3) Short proofs (you may be asked to reproduce them).

**B.** Some items to know:

- (1) Definition of group and field, uniqueness of identities and inverses, cancellation
- (2) Definition of vector space, how to tell whether a given set and operations form a vector space
- (3) Immediate consequence of the vector space definition: cancellation, uniqueness of zero and inverses, multiplication by scalar or vector zero gives result zero, inverse commutes strongly with scalar multiplication
- (4) Definition of subspace, how to tell whether a given subset of a vector space is a subspace
- (5) Standard examples of subspaces of  $M_{n \times n}$ : diagonal and symmetric matrices
- (6) Result of union or intersection of subspaces
- (7) Definition of linear combination, span, generate
- (8) How to set up and solve a system of linear equations given a linear combination of vectors with some unknowns
- (9) Definition of linear dependence, linear independence, relationship to linear combinations and span
- (10) How to use linear equations to determine whether a set of vectors is linearly independent
- (11) What linear dependence and independence of one set  $S_1 \subset S_2$  tells you about the other set (if anything)
- (12) Definition of basis; relationship of basis size to size of linearly independent sets and spanning sets, number of representations of a vector of  $V$  as linear combinations of its basis vectors
- (13) The standard bases for the key examples in A2 above (except  $\mathcal{F}(S, F)$ )
- (14) How to obtain a basis from a spanning set; the replacement theorem and its corollary about basis size
- (15) Definition of dimension, dimension of subspace

**C.** Some items not to memorize:

- (1) The definition of characteristic for a field; that will not be on the exam.
- (2) The definitions of trace,  $+$  (sum of vector spaces), and  $\oplus$ : if you need them I will provide their definitions.
- (3) Proofs. Even the short ones – memorizing them is a worse use of your time than simply trying to understand them and memorizing the usual tricks (adding 0 in some form, for instance).
- (4) Lagrange interpolation: we skipped this entirely.

**D.** Some ways to practice, if you're looking for more work:

- (1) Invent collections of vectors in the standard vector space examples (where the field is  $\mathbb{R}$  or  $\mathbb{C}$  and calculate whether they are linearly dependent or independent.
- (2) Look at problems in the book similar to homework exercises: if I gave you some of the parts of a problem, try other parts. In particular, 1.2 #10–12, 16, 18–21; 1.3 #10–13, 22; 1.4 #5, 12, 16; 1.5 #8a, 10, 12–15, 17; 1.6 #4, 8, 9, 13, 14, 16, 17, 21
- (3) Try to write short proofs in your own words.