Coordinates: New Axes

In $\mathbb{R}^2$ it is relatively easy to visualize a basis as determining a set of coordinate axes, the lines that run through $(0,0)$ and the vectors of the basis. The entries of the $\mathcal{B}$-coordinate vector of $\boldsymbol{x}$ are the distances one must proceed from the origin parallel to each axis.

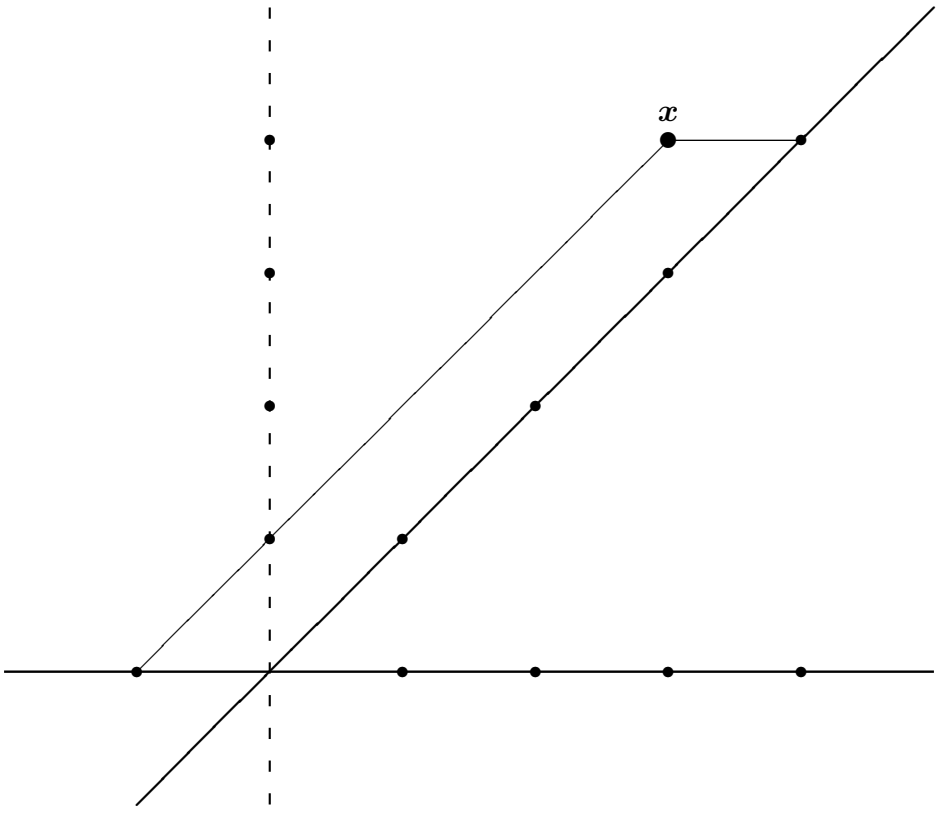
Careful: the unit of measurement is one *basis vector length*, not one standard unit. I'll point that out in the examples.

Back to our example from earlier in lecture:

$$\boldsymbol{x} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \quad \mathcal{B}_2 = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}, \quad [\boldsymbol{x}]_{\mathcal{B}_2} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}.$$

The axis system: solid lines are the $\mathcal{B}_2$ axes, dashed line is the standard axis that isn't a $\mathcal{B}_2$-axis, dots appear on the axes every unit distance (as determined by the length of the basis vectors: so on the standard axes they are one standard unit apart, and on the nonstandard axis they are $\sqrt{2}$ standard units apart).

To find $\boldsymbol{x}$, we walk -1 units west and 4 basis vector units ($4\sqrt{2}$ standard units) northeast, in either order.



$$\boldsymbol{x} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \quad \mathcal{B}_3 = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}, \quad [\boldsymbol{x}]_{\mathcal{B}_2} = \begin{bmatrix} 2/3 \\ 5/3 \end{bmatrix}.$$

The axis system: solid lines are the $\mathcal{B}_3$ axes, dashed lines are the standard axes, dots appear on the axes every unit distance (as determined by the length of the basis vectors: so on the standard axes they are one standard unit apart, and on the nonstandard axes they are $\sqrt{5}$ standard units apart).

To find $\boldsymbol{x}$, we walk $\frac{2}{3}$ of a basis vector unit ($\frac{2}{3}\sqrt{5}$ standard units) along the line with slope $\frac{1}{2}$ and $\frac{5}{3}$ of a basis vector unit ($\frac{5}{3}\sqrt{5}$ standard units) along the line with slope 2.