Math 20, Spring 2005, Test 2

Instructions: Problems 1–7 count 12 points each, while the last problem counts 16 points. You may use a calculator to help with arithmetic, including logs, exponentiation, and factorials (if there is a factorial button).

1. A typesetter makes on average 1 typographical error per 1000 words. A book he is typesetting has on average 300 words per page and is 500 pages long. What is the probability that a random page has 2 or more errors?

2. In problem 1, how many errors and how many error-free pages would one expect in the book?

3. An urn contains 2 gold balls and 3 silver balls. You draw balls at random without replacing them until you've drawn both of the gold balls. Each time you draw a ball you win a dollar if it is gold and lose a dollar if it is silver. What is the expectation for this game?

4. Alice and Bob play "heads and tails" (I'm not making this up, it's in the book) where a fair coin is fairly flipped n times. Each time it comes up heads, Alice wins a penny from Bob, and each time it comes up tails, she loses a penny to Bob. Let A be Alice's winnings (which may be negative if she loses money). Find $E(A)$ and $V(A)$.

5. What is Chebyshev's inequality? How is it proved?

6. A fair coin is fairly flipped 10,000 times. What is the approximate probability that it lands heads exactly 4971 times?

7. In problem 6, what is the approximate probability that the coin lands heads fewer than 4971 times?

8. Consider a lottery where there is a 0.9 chance of not winning anything, a 0.099 chance of winning $5, and a 0.001 chance of winning $250. The lottery ticket costs $1. What is the expected value of this game? What is the probability of breaking even or better if you buy 100 tickets?