Math 20, Spring 2005, Test 1

Instructions: Problems 1–7 count 10 points each, while the last two problems count 15 points each. You may use a calculator to help with arithmetic, including logs, exponentiation, and factorials (if there is a factorial button).

1. You and a friend are in a group of 10 people. Five are randomly chosen for kitchen duty. What is the probability that you are both chosen?
2. Suppose that A, B are independent events on a sample space. Prove that $\tilde{A}, B$ are also independent.
3. Suppose A_1, A_2, A_3, A_4 are events, each with probability 0.3 of occurring, each pairwise intersection has probability 0.2 of occurring, each triple intersection has probability 0.1 of occurring, and the 4 events can never simultaneously occur. Find the probability that at least one of the events occurs.
4. A state has license plates consisting of 3 letters followed by 3 digits. If there are no forbidden combinations, how many license plates are possible? (As usual, different orderings of the same characters correspond to different license plates.)
5. In a card game a player has figured out that two opponents have between them 6 hearts in their hands. Assuming all possibilities are equally likely, what is the probability that these hearts are split $3, 3$? What is the conditional probability for this event if you know that they each have at least one heart?
6. What is Stirling's formula?
7. What is the exact probability of getting at least 60 heads when tossing a fair coin 100 times? Is this probability more or less than one chance in 60?
8. A sample space Ω conists of the 9 ordered triples (i, j, k) where i, j, k are integers in $\{1, 2, 3\}$ and are either all different or all the same.
   (a) List the 9 members of Ω.
   (b) Let A_1 be the event that a 2 is in the first coordinate, A_2 is the event that 2 is in the second coordinate, and A_3 is the event that 2 is in the third coordinate. Compute $P(A_1), P(A_2), P(A_3)$.
   (c) Show that the events A_1, A_2, A_3 are pairwise independent (each pair is independent), but they are not mutually independent.
9. It has been discovered that 20% of major league baseball players use steroids. A certain drug test gives a positive for steroid use 99% of the time if you are using steroids, and 2% of the time if you are clean.
   (a) What is the probability of a random player testing positive?
   (b) What is the probability of being a steroid user if you test positive?
   (c) Suppose you don't know the prevalence of steroid use, but in a large random sample you find 10% testing positive. What would be a reasonable estimate for the percentage using steroids?