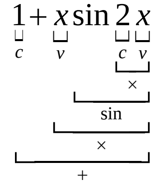
Variables and expressions

- Once we recognize a pattern, we'd like to write it in its most general form; **variables** are what allow us to do this in a concise and precise manner, by acting as *placeholders*.
  – e.g., instead of ambiguously writing "$2+1=1+2,\ 3+1=1+3,\ 3+2=2+3,\ldots$",
  we can simply say "$a+b=b+a$ for all numbers a and b".

- Two things to keep in mind when using variables:
  ① *Never* use one variable to represent more than one thing in the same context.
  ② Equal quantities can be substituted for each other at will; but be sure to:
  – substitute for all occurrences consistently
  – use $(\,)$; e.g., changing $[n \to n+1]$ in "$3n$" gives $3(n+1)=3n+3$.

- **Expressions** are built from…
  ▫ constants: 0, 1, e, π, …
  ▫ variables: $x, y, t, \theta, \ldots$
  ▫ functions: sin, cos, ln, exp, …
  ▫ operations: +, −, ×, ÷, powers, …

- When we write an expression, its structure is obscured; it is *crucial* to be able to correctly **parse** a written expression determine its structure! e.g.: $1+x\sin 2x$



Functions and graphs

- A **function** is a *rule* that assigns to each element in the *domain* just one element of the *range*.
  – when functions are described via expressions, such as $f(x)=x^2-\sin x$,
  the variable used is entirely irrelevant — for example, "$f(\xi)=\xi^2-\sin\xi$" describes the same function f.

- The **graph** of a function f consists of all points $(x, f(x))$ with x in the domain of f.
  – the graph of a function can be used to determine its domain, range, values, asymptotes, etc.

- Starting from a known graph, say $y=g(x)$, we can build the graphs of related functions such as $y=2-g(3x-1)$ by carefully parsing the new expression and manipulating the original graph.