1. Find the centroid of the top half of the region enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

2. Let $S$ be the boundary of the solid region $R$ in $\mathbf{R}^3$ bounded by the paraboloids $z = 18 - x^2 - y^2$ and $z = x^2 + y^2$. Find the surface integral of $\mathbf{F} = (0, 0, z^2)$ over $S$, where $S$ is oriented by outward-pointing normals.

3. Let $C$ be the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$, oriented counterclockwise. Find the line integral $\oint_C \mathbf{F} \cdot d\mathbf{s}$, where $\mathbf{F} = (\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2})$. (Note: Green's theorem does not hold for the entire region within $C$ since the vector field is not defined throughout this region.)

4. For each of the two vector fields below, either find a function $f$ such that $\mathbf{F} = \nabla f$, or explain why no such function $f$ exists.
   (a) $\mathbf{F}(x, y, z) = y^2\mathbf{i} + 2xy\mathbf{j} + xz\mathbf{k}$
   (b) $\mathbf{F}(x, y) = (3x^2y + y^2)\mathbf{i} + (x^3 + 2xy)\mathbf{j}$
   (c) Evaluate $\int_{\mathbf{x}} y^2\, dx + 2xy\, dy + xz\, dz$, where $\mathbf{x}$ is the line segment from $(0, 0, 0)$ to $(1, 1, 1)$.
   (d) Let $\mathbf{F}$ be as in part (b). Find the line integral of $\mathbf{F}$ along the portion of the circle $r = \sqrt{2}$, oriented clockwise, from $(\sqrt{2}, 0)$ to $(-1, 1)$.

5. Evaluate $\iiint_S z\, dV$ where $S = \{(x, y, z) : a^2 \leq x^2 + y^2 + z^2 \leq b^2,\ z \geq 0\}$ is the region bounded by the plane $z = 0$ and the top halves of the spheres of radius $a$ and $b$. (Assume $a < b$.)