7.1 (a) $\Sigma(s,t) = (s\cos t, s\sin t, s^2)$

7 $T_s(s,t) = (\cos t, \sin t, 2s)$

$T_t(s,t) = (-s\sin t, s\cos t, 0)$.

$N = T_s \times T_t = (-2s^2\cos t, 2s^2\sin t, s) \neq 0$ if $s \neq 0$.

$\neq 0$. if $s = 0$ and $t \neq 0$.

So not smooth at $(0,0,0)$.

(b) Note $z = x^2 + y^2$ so it's a paraboloid.

11 (a) $f(x,y) = \sqrt{4 - (x-2)^2 - (y-1)^2}$

$\frac{\partial f}{\partial x} = \frac{-(x-2)}{\sqrt{\quad}}$ $\quad \frac{\partial f}{\partial y} = \frac{-(y+1)}{\sqrt{\quad}}$

$\frac{\partial f}{\partial x}\Big|_{(1,0,\sqrt{2})} = \frac{1}{\sqrt{2}}$ $\quad \frac{\partial f}{\partial y}(1,0,\sqrt{2}) = -\frac{1}{\sqrt{2}}$.

tangent plane $\quad z = f(1,0) + \frac{\partial f}{\partial x}(1,0)(x-1)$

$+ \frac{\partial}{\partial x}(1,0)\, y$

or $-x + y + \sqrt{2}z = 1$

(b) $\nabla F = 2(x-2, y+1, z)$ $\quad \nabla F(1,0,\sqrt{2}) = (-2, 2, 2\sqrt{2})$

$\therefore \quad 0 = \nabla F(1,0,\sqrt{2}) \cdot ((x,y,z) - (1,0,\sqrt{2}))$

$= (-2, 2, 2\sqrt{2}) \cdot ((x,y,z) - (1,0,\sqrt{2}))$

$\Leftrightarrow -x + y + \sqrt{2}z = 1$

(c) $\cos s = \frac{\sqrt{2}}{2}$ & $2\sin s \sin t = 1$ & $\sin s\cos t = -1 \Rightarrow s = \frac{\pi}{4}$ $t = \frac{3\pi}{4}$

$N(\pi/4, 3\pi/4) = T_s(\pi/4, 3\pi/4) \times T_t(\pi/4, \frac{3\pi}{4}) = (-\sqrt{2}, \sqrt{2}, 2)$

and so on.

20) Parameterization $\Sigma(s,t) = (s\cos t, s\sin t, 9-s^2)$

$0 \le s \le 3$, $0 \le t \le 2\pi$

$T_s(s,t) = (\cos t, \sin t, -2s)$ $T_t(s,t) = (-s\sin t, s\cos t, 0)$

$\|T_s \times T_t\| = s\sqrt{4s^2+1}$.

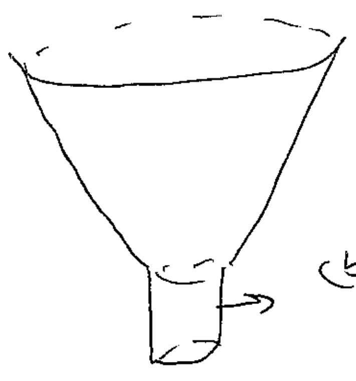
$S.A = \int_0^{2\pi}\int_0^3 s\sqrt{4s^2+1}\,ds\,dt = \frac{\pi}{6}\left(37^{3/2}-1\right)$.

7.2 19) $\iint_S (-y\,i + x\,j - k)\cdot n\,dS = -\iint_S k\cdot n\,dS - \iint_S (y\,i - x\,j)\cdot n\,dS$

$= -\iint_S k\cdot n\,dS$

$= -\int_0^{2\pi}\int_0^{\frac{\pi}{2}} (0,0,1)\cdot(a^2\sin^2\varphi\cos\theta, a^2\sin^2\varphi\sin\theta, a^2\sin\varphi\sin\varphi)\,d\varphi\,d\theta$

$= -\pi a^2$

21) a) 

cylindrical part $\hat{n}(x,y) = (x,y)$
$= (\cos t, \sin t)$

(b) conical part $\Sigma(s,t) = (t\cos s, t\sin s, t)$

$T_s = (t\sin s, t\cos s, 0)$ $T_s \times T_t = (t\cos s, t\sin s, -t)$

$T_t = (\cos s, \sin s, 1)$ $\Rightarrow \hat{n} = \frac{1}{\sqrt{2}}(\cos s, \sin s, -1)$.

(c) $\iint_S F\cdot dS = \iint_{conical} + \iint_{cylindrical}$

$= \int_0^{2\pi}\int_0^1 (-\sin s, \cos s, t)\cdot(\cos s, \sin s, 0)\,dt\,ds + \int_0^{2\pi}\int_1^9 (t\sin s, t\cos s, t)\cdot(t\cos s, t\sin s, -t)\,dt\,ds$

$= \frac{-1456\pi}{3}$.

7.3

11) $\nabla \times F = (x^2 + e^x y \sin(yz))\hat{\imath} + 5\hat{\jmath} + (e^x \cos(yz) - 2xz)\hat{k}$

$\iint_S \nabla \times F \cdot dS = \iint_{S_1} \nabla \times F \cdot dS$ where S_1 = disk in the plane $y=1$ bounded by $x^2+z^2=9$

~~n has~~ $n \cdot S_1$ has normal $\langle 0,1,0 \rangle$

$$\iint_S \nabla \times F \cdot dS = \iint_{S_1} \nabla \times F \cdot dS$$

$$= \iint_{S_1} (\nabla \times F) \cdot \langle 0,1,0 \rangle \, dS$$

$$= \iint 5 \, dS = 5 \, \pi 3^2 = 45\pi$$

15) Check: If $F = \nabla f$, then $\nabla \times F = 0$.

Suppose $\nabla \times F = 0$ and C is any closed, simple curve. We show $\oint_C F \cdot ds = 0$. Let S be a surface whose boundary is C, oriented consistently with C. Then

$$\oint_C F \cdot dS \underset{\text{by Stokes}}{=} \iint_S \nabla \times F \, dS \underset{\text{since } \nabla \times F = 0}{=} \iint 0 \, ds = 0.$$

5) ∂S : $\sigma(t) = (3\cos t, 3\sin t)$

$$\therefore \oint_{\partial S} F \cdot ds = \int_0^{2\pi} F(\sigma(t)) \cdot \sigma'(t) \, dt$$

$$= \int_0^{2\pi} (27\cos^3 t, 3^7 \sin^7 t) \cdot (-3\sin t, 3\cos t) \, dt$$

$$= 0.$$

7.4 (a) Let $\vec{a} = (a_1, a_2, a_3)$ $\sigma(t) = (x(t), y(t), (z(t)))$

13) $\oint_C \vec{a} \cdot ds = \int_a^b \vec{a} \cdot \sigma'(t)\, dt$

$= \int_a^b a_1 x'(t) + a_2 y'(t) + a_3 z'(t)\, dt$

$= a_1 x(t) + a_2 y(t) + a_3 z(t) \Big|_a^b$

Since C is closed $x(a) = x(b)$ so

$= 0.$

(b) $\oint a \cdot ds = \iint_S \nabla \times a \cdot dS = \iint 0 \, dS = 0.$

16 For gauss's thm to apply, S must be closed. Hence ∂S is empty. Thus $\oint_{\partial S} G \cdot ds$ is not defined.