Math 13 Fall 2009 practice exam

1. Set up the integral (do not evaluate) for the mass of the solid between the two parabolas $z = 12 - x^2 - y^2$ and $z = 2(x^2 + y^2)$ where the density at any point is the distance of the point to the origin (use cylindrical coordinates).

2. Set up the integrals (do not evaluate) to compute the x-coordinate of the center of mass of the solid inside the ball $x^2 + y^2 + z^2 \leqslant 1$ outside the cone $z^2 = x^2 + y^2$ where the density is given by $\rho(x, y, z) = e^x$ (use spherical coordinates).

3. Use the change of coordinates $u = x + 2y$ and $v = 2x - y$ to solve the integral

$$\iint_D \frac{(2x - y)^2}{(x + 2y)^4} dA$$

where D is the region between the lines $x + 2y = 2$, $x + 2y = 4$, $2x - y = 1$ and $2x - y = 3$.

4. A garden is bounded by a fence along $y = x^3$ for $1 \leqslant x \leqslant 5$. The height of the fence is given by $f(x, y) = x^3$ (measurements in meters). If 1 Liter of paint is good for $10m^2$ how much paint do you need to paint the fence (on one side)?

5. For each of the following check whether the vector field F is conservative and if it is find a potential function.
a) $F(x, y) = \langle 2xe^y + 2xy, x^2 e^y + x^2 + y \rangle$
b) $F(x, y) = \langle 2xe^y + 3xy, x^2 e^y + x^2 \rangle$

6. Find the work done by a force F moving a particle along the path c.
a) $F(x, y) = \langle x - y, x \rangle$ and c is the part of the circle $x^2 + y^2 = 4$ where $y \geqslant 0$ traversed counterclockwise.
b) $F(x, y) = \langle 2x + y, \tan(y^3) + x \rangle$ and c is the circle $x^2 + y^2 = 1$ traversed counterclockwise starting at $(1, 0)$.

7. Find the integral of the function $f(x, y, z) = x^2 + y^2 + z^2$ over the region which is bounded by the spheres of radius 5 and 3 and which is above

the plane $z = 0$.

8. Find the volume of the solid that lies within both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 1$.