Math 13 Fall 2009 Homework 7

1.) Given a vector field $\mathbf{F}(x, y) = (P(x, y), Q(x, y))$, a parameterized curve $\mathbf{r}(t)$ with $a \le t \le b$ is called an integral curve of $\mathbf{F}$ if $\mathbf{r}'(t) = \mathbf{F}(\mathbf{r}(t))$ for $a \le t \le b$.

a.) Show that the vector field $\mathbf{F}(x, y) = (-9y, x)$ is not conservative.

b.) Prove that $\mathbf{r}(t) = (3\cos(3t), \sin(3t))$ is an integral curve of $\mathbf{F}$ for $0 \le t \le \frac{2}{3}\pi$.

c.) Calculate the work done by the vector field $\mathbf{F}$ on a particle traveling along the curve $\mathbf{r}(t) = (3\cos(3t), \sin(3t))$ from $0 \le t \le \frac{2}{3}\pi$.

d.) Calculate the work done by the vector field $\mathbf{F}$ on a particle traveling along the curve $\mathbf{r}(t) = (3\cos(3t), 3\sin(3t))$ from $0 \le t \le \frac{2}{3}\pi$.

e.) Are the integrals from (c) and (d) equal? Explain why or why not.

2.) Consider the vector field $\mathbf{G}(x, y) = (1, 1)$.

a.) Show that $\mathbf{G}$ is conservative, and find scalar potential function of $\mathbf{G}$.

b.) Find an integral curve $\mathbf{r}(t)$ of $\mathbf{G}$ such that $\mathbf{r}(0) = (0, 1)$ for $0 \le t \le 1$. What is the point $\mathbf{r}(1)$?

c.) What is the line integral of the vector field along the integral curve you found in part (b)?