

Math 13 Fall 2009 Homework 3. Due Friday October 16th in class.

1.) Suppose that the function $f(x, y) = 20 - x^2 - 3y^2$ represents the elevation of the terrain near the point on the earth: latitude $63^\circ 04' 10''$ north, longitude $151^\circ 0' 26''$ west. Units for x, y and $f(x, y)$ are in thousands of feet, and they y axis represent 'north'.

- a. What mountain is actually at this point? Give reference to your source.
- b. Describe and parameterize the level curves of f corresponding to values 19, 18, 17, 16, 15 and 14. What do these curves represent on the mountain?
- c. Plot these level curves on a piece of graph paper using at least 3 hashes to each unit in the x and y directions. What is the name of the type of map you have created? Mark the summit of the mountain. What is the (x, y) coordinate of the summit? What is its elevation?
- d. What is the elevation at the point $(1, \sqrt{\frac{4}{3}})$? Plot this point on the same graph paper as above.
- e. In what direction should you head to ascend most quickly? Reference any facts you use from class or the text. Describe the direction in **degrees** counterclockwise from east. Reference any facts you use from class or the text.
- f. In what direction should you head to descend most quickly? Describe the direction in **degrees** counterclockwise from east. Reference any facts you use from class or the text.
- g. In what direction should you head to stay at the same elevation with the uphill direction on your left? Describe the direction in **degrees** counterclockwise from east. Reference any facts you use from class or the text.
- h. What is the ∇f at the summit? Interpret the physical significance of this mathematical fact.

2.) Let $f(x, y) = \cos(\frac{\pi}{6}(x^2 + y^2))$ and let $P = (1, \sqrt{3})$.

- a. Find the equation of the tangent plane of f at P .
- b. Find a parametrization of the level curve of f through P and compute its

arclength.

c. Compute the directional derivative of f at P in the direction pointing towards the origin.

d. Find the directional derivative of f at P in the direction $u = \langle -\frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$. Explain why you would expect this answer.