Math 13 Fall 2009 Homework 10

1. Let $z = f(x,y) = -(x^2+y^2)^2 + 3(x^2+y^2) + 4$ and $z = g(x,y) = x^2+y^2-4$.
a) Compute the intersection of the graphs of the two functions in $\mathbb{R}^3$. We will denote this curve by $\gamma(t)$.
b) Compute the volume between the graphs.
c) Find the local maxima and minima of f (there is just one).
d) Compute the angle of intersection between them at the point $(2,0,0)$. Hint: The angle between the surfaces is the angle between the tangent planes.
e) Let $c(t) = g(\sqrt{3}t, t^2)$. Compute the intersection of $c(t)$ with the graph of f. This becomes a lot easier if you use what you found in part a).
d) Use the chain rule to find $\frac{\partial}{\partial t}c(t)$. Set up the integral to find the arclength of $c(t)$ between the two intersections points, then use a computer to find a numeric approximation to the length.
f) Let $F(x,y,z) = \langle \sin(x) + e^{z^2}, x(z^3+1), 2^{\sin(x)}3^{\cos(z)} \rangle$. Use Stokes theorem to compute the integral of $\int_\gamma (F \circ \gamma)(t) \cdot \gamma'(t)dt$. Choose the surface you integrate over very carefully.
g) Let $G(x,y,z) = \langle x^3, y^3, z^3 \rangle$. Use the divergence theorem to compute the integral of G over the surface f.