Math 13 Fall 2009 Homework 1

Due Friday October 2, 2009 in class.

1) Show that the area of the parallelogram spanned by $\overrightarrow{a}$ and $\overrightarrow{b}$ is given by: $\sqrt{||\overrightarrow{a}||^2||\overrightarrow{b}||^2 - (\overrightarrow{a} \cdot \overrightarrow{b})^2}$ Show that the formula holds both for vectors in $\mathbb{R}^2$ and in $\mathbb{R}^3$, depending on how you approach the problem you might have to treat the two cases seperatly.

2) Show that any three vectors $\overrightarrow{a}$, $\overrightarrow{b}$ and $\overrightarrow{c}$ satisfy:

$$\overrightarrow{a} \times (\overrightarrow{b} \times \overrightarrow{c}) = (\overrightarrow{a} \cdot \overrightarrow{c})\overrightarrow{b} - (\overrightarrow{a} \cdot \overrightarrow{b})\overrightarrow{c}$$

3.) Page 839, Number 64

Find the parametric equations for the line through the point $(0, 1, 2)$ that is perpendicular to the line $x = 1 + t$, $y = 1 - t$, $z = 2t$ and intersects this line.

4.) Page 821, Number 42

The vector $orth_{\mathbf{a}}\mathbf{b} = \mathbf{b} - proj_{\mathbf{a}}\mathbf{b}$ is called the Orthogonal Projection of $\mathbf{b}$ with respect to $\mathbf{a}$. It is not hard to check that $orth_{\mathbf{a}}\mathbf{b}$ is orthogonal to $\mathbf{a}$ (see problem 41 on page 821 in the text). For the vectors $\mathbf{a} = \langle 1, 2 \rangle$ and $\mathbf{b} = \langle -4, 1 \rangle$ find $orth_{\mathbf{a}}\mathbf{b}$ and illustrate (the relationship between these vectors) by drawing the vectors $\mathbf{a}$, $\mathbf{b}$, $proj_{\mathbf{a}}\mathbf{b}$ and $orth_{\mathbf{a}}\mathbf{b}$.