Math 112
Introduction to Riemannian Geometry
Spring 2006
Assignment 4
Due May 9, 2006

Chp. 4 (do Carmo): 1, 2, 3, 4, 6, & 7

1. Show that the geodesics γ of $\mathbb{C}P^n$ with $\gamma(0) = \pi(\tilde{x}) = x$ are are of the form $t \mapsto \pi((\cos t)\tilde{x} + (\sin t)\tilde{v})$, where
$$\pi : (S^{2n+1}, g_{\text{std}}) \to (\mathbb{C}P^n, h_{\text{std}})$$
is the canonical Riemannian submerion and $\tilde{v} \in T_{\tilde{x}}S^{2n+1} \leq T_{\tilde{x}}\mathbb{C}^{n+1}$ is orthogonal to $\tilde{x}$ and $i\tilde{x}$.

2. Let M be a manifold with connection ∇. If Y is a vector field on M and $c : I \to M$ is a curve such that $c'(0) = v \in T_pM$, then $\nabla_v Y$ depends only on the values of Y along c. That is if X is another vector field and $X \circ c = Y \circ c$, then $\nabla_v X = \nabla_v Y$.