MATH 81/111: RINGS AND FIELDS
HOMEWORK #5

Problem 5.1. Is the field of constructible numbers Galois over $\mathbb{Q}$? Give a proof or a counterexample.

Problem 5.2. Let $a \in \mathbb{Q}$, and let $f(X) = X^3 + aX^2 - (a+3)X + 1 \in \mathbb{Q}[X]$ be irreducible.

(a) Show that f has Galois group $\mathbb{Z}/3\mathbb{Z}$.

(b) Let $F = \mathbb{Q}(\alpha)$ with α a root of f. Show that the map $\alpha \mapsto 1/(1-\alpha)$ generates $\mathrm{Gal}(F/\mathbb{Q})$.

Problem 5.3. Let $D(f)$ be the discriminant of a polynomial $f \in F[X]$.

(a) Show that
$$D(X^n - 1) = (-1)^{\binom{n}{2}+n-1} n^n.$$

(b) Let $h \in F[X]$ be monic and let $g(X) = (X-a)h(X)$ with $a \in F$. Show that
$$D(g) = h(a)^2 D(h).$$

(c) Show that
$$D(X^{n-1} + X^{n-2} + \cdots + 1) = (-1)^{(n-1)(n+2)/2} n^{n-2}.$$

Date: 4 February 2015; due Wednesday, 11 February 2015.