MATH 81/111: RINGS AND FIELDS
HOMEWORK #3

Problem 3.1 (M2-5). Let $f \in F[X]$ where F is a field of characteristic zero. Let $d = \gcd(f, f')$. Show that $g = f/d$ has the same roots as f and that all roots of g are simple.

Problem 3.2 (M2-6). Let $f \in F[X]$ be irreducible where F has characteristic p. Show that $f(X) = g(X^{p^e})$ for some $e \geq 0$ and $g(X)$ is irreducible and separable. Deduce that every root of $f(X)$ has the same multiplicity p^e in any splitting field.

Problem 3.3. Let $F \subseteq K \subseteq L$ be a tower of fields. Suppose that $\alpha \in L$ is separable over F. Show that α is separable over K. Conclude (or argue otherwise) that if $f \in F[X]$ is separable then any splitting field of f is separable over F.

Problem 3.4. Let $f = X^5 + X + 1 \in \mathbb{Z}[X]$. For a prime p, let $f_p \in \mathbb{F}_p[X]$ denote the reduction of f modulo p. Find all primes p for which f_p is not separable, and for such primes compute $\gcd(f_p, f_p')$.

Date: 21 January 2015; due Wednesday, 28 January 2015.