MATH 105: HW #4

(1) Let T_n denote the nth triangular number: $T_n = n(n+1)/2$. It is called this since it is the sum of $1, 2, \ldots, n$, which can be thought of as T_n pebbles arranged in a triangle, with j stones on level j. Use the version of Hypothesis H given in class to show that T_n+1 is prime infinitely often. (Note that $f(x) = x(x+1)/2 \notin \mathbb{Z}[x]$. Hint: Consider separately n odd or n even.)

(2) Use the version of Hypothesis H given in class to show that if $f(x) \in \mathbb{Q}[x]$ has the properties
   (a) For each $n \in \mathbb{Z}$, we have $f(n) \in \mathbb{Z}$,
   (b) f is irreducible,
   (c) the leading coefficient of f is positive, and
   (d) for each prime p, there is some integer n with $p \nmid f(n)$,

   then there are infinitely many integers n with $f(n)$ prime.

(3) Also do problems 31 and 32 in Chapter 1 of the book.