MATH 101: ALGEBRA I
HOMEWORK, DAY #25

Problem 25.1. Let M be the $\mathbb{Z}$-module generated by x_1, x_2, x_3, x_4 subject to the relations

$$\begin{aligned} x_1 + 3x_2 - 9x_3 \phantom{{}+12x_4} &= 0 \\ x_1 + 3x_2 + 3x_3 + 12x_4 &= 0 \\ 2x_1 + 4x_2 + 2x_3 + 24x_4 &= 0 \end{aligned}$$

Give an explicit isomorphism of M to a direct sum of cyclic abelian groups. What are the invariant factors and elementary divisors of $\operatorname{Tor}(M)$?

Problem 25.2. Let R be a PID. Let $a, b \in R$, not both zero. Write $(a, b) = (g)$ for $g \in R$, so that there exist $x, y \in R$ such that $ax + by = g$. Show that $(x, y) = R$.

Date: Assigned Friday, 3 November 2017; due Monday, 6 November 2017.