MATH 101: GRADUATE LINEAR ALGEBRA
DAILY HOMEWORK #19

Let R be a commutative ring.

Problem 19.1. Let $S \subseteq R$ be a multiplicatively closed subset. Consider the relation $\sim$ on $R \times S$ by $(r, s) \sim (r', s')$ if there exists $x \in S$ such that $x(rs' - r's) = 0$. Show that $\sim$ is transitive. (What happens if you remove the x?)

Problem 19.2. Let $f \in R$ be not nilpotent. Let $S = \{f^k : k \geq 0\}$. Consider the ring $R[x]/(fx - 1)$, the quotient of the univariate polynomial ring $R[x]$ by the ideal $(fx - 1)$. Show that $R[S^{-1}] \simeq R[x]/(fx - 1)$ as rings. (What happens if f is nilpotent?)

Date: Assigned Monday, 23 October 2017; due Wednesday, 25 October 2017.