

MATH 101: GRADUATE LINEAR ALGEBRA
DAILY HOMEWORK #17

Problem 17.1. Let R be a ring and let

$$\begin{array}{ccccccccc} 0 & \longrightarrow & M & \xrightarrow{\psi} & N & \xrightarrow{\phi} & Q & \longrightarrow & 0 \\ & & \downarrow \alpha & & \downarrow \beta & & \downarrow \gamma & & \\ 0 & \longrightarrow & M' & \xrightarrow{\psi'} & N' & \xrightarrow{\phi'} & Q' & \longrightarrow & 0 \end{array}$$

be a homomorphism of short exact sequences. Suppose that α and γ are surjective. Show that β is surjective.

Problem 17.2. Let R be a ring and let

$$0 \longrightarrow M \xrightarrow{\psi} N \xrightarrow{\phi} Q \longrightarrow 0$$

be a short exact sequence. Let D be an R -module. Show that the map

$$\mathrm{Hom}_R(D, M) \rightarrow \mathrm{Hom}_R(D, N) \rightarrow \mathrm{Hom}_R(D, Q)$$

is exact. [*Hint: This is Theorem 28 on page 387 in Dummit and Foote.*]