MATH 101: GRADUATE LINEAR ALGEBRA
DAILY HOMEWORK #16

Problem 16.1. Let R be a commutative ring.

(a) Let A, B be R-algebras (with 1). Show that there exists a unique structure of R-algebra on the R-module $A \otimes_R B$ such that

$$(\alpha \otimes \beta) \cdot (\alpha' \otimes \beta') = \alpha\alpha' \otimes \beta\beta'$$

for all $\alpha, \alpha' \in A$ and $\beta, \beta' \in B$.

(b) Let A be an R-algebra, let S be a commutative ring, and let $f\colon R \to S$ be a ring homomorphism. Equip S with the structure of R-module via f. (Write out what this means.) Show accordingly that $A \otimes_R S$ can be given the structure of an S-algebra.

(c) Describe the $\mathbb{C}$-algebras $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{C}$ and $\mathbb{C} \otimes_{\mathbb{C}} \mathbb{C}$.

Date: Assigned Monday, 16 October 2017; due Wednesday, 18 October 2017.