

MATH 101: GRADUATE LINEAR ALGEBRA
DAILY HOMEWORK #13

Problem 13.1. Let R be a commutative ring. Show that the map $R \rightarrow \text{End}_R(M)$ where $r \in R$ maps to the multiplication-by- r endomorphism

$$\begin{aligned}\phi_r : M &\rightarrow M \\ m &\mapsto rm\end{aligned}$$

is a ring homomorphism, and thereby that $\text{End}_R(M)$ has the structure of an R -algebra. (What happens if R is not commutative?)

Problem 13.2. Let R be commutative ring.

- (a) Prove that $\text{Hom}_R(R, M) \simeq M$ as R -modules.
- (b) Consider R as an R -module. Prove that there is a ring isomorphism $\text{End}_R(R) \simeq R$.