MATH 101: GRADUATE LINEAR ALGEBRA
DAILY HOMEWORK #12

Problem 12.1. Let M be the $\mathbb{Z}$-module $\mathbb{Z}/24\mathbb{Z} \times \mathbb{Z}/15\mathbb{Z} \times \mathbb{Z}/50\mathbb{Z}$.

(a) Find $\operatorname{ann}(M)$, the annihilator of M in $\mathbb{Z}$.
(b) Let $I = 2\mathbb{Z}$. Compute the annihilator of I in M.

Let R be a ring (with 1). We say $x \in R$ is a *left zerodivisor* if $x \neq 0$ and there exists a nonzero $y \in R$ such that $xy = 0$. We similarlydefine *right zerodivisor*. If R is commutative, there is no difference between left and right, so we simply say *zerodivisor*.

We say R is a *domain* if R is commutative and has no zerodivisors.

Problem 12.2. Let R be a ring, and let M be left R-module. An element $m \in M$ is called a *torsion element* if $rm = 0$ for some nonzero $r \in R$. The set of torsion elements is denoted $\operatorname{Tor}(M)$.

(a) Prove that if R is a domain, then $\operatorname{Tor}(M)$ is a submodule of M.
(b) Give an example of a ring R and an R-module M such that $\operatorname{Tor}(M)$ is not a submodule.
(c) Show that if R has a zerodivisor then every nonzero R-module M has $\operatorname{Tor}(M) \neq \{0\}$.
(d) M is called a *torsion module* if $M = \operatorname{Tor}(M)$. Prove that every finite abelian group is a torsion $\mathbb{Z}$-module. Give an example of an infinite abelian group that is a torsion $\mathbb{Z}$-module.

Date: Assigned Friday, 6 October 2017; due Monday, 9 October 2017.