MATH 101: ALGEBRA I
HOMEWORK, DAY #15

Problem JV15.A. Let R be a commutative ring. Let G be a monoid, written multiplicatively. Define the additive group

$$R[G] := \bigoplus_{g \in G} R = \left\{ \alpha = \sum_{g \in G} a_g[g] : a_g = 0 \text{ for all but finitely many } g \right\}.$$

Define a product on $R[G]$ by $[g][h] = [gh]$, extending by distributivity. Then $R[G]$ is an R-algebra, with multiplicative identity $[1]$ (check this if you need to!) called the *monoid algebra* of G over R.

(a) The polynomial ring $R[x_1, \dots, x_n]$ is a monoid ring: for what monoid?

(b) Let $G = S_3$ and $R = \mathbb{Z}$. Let

$$\alpha = 3(1\ 2) - 5(2\ 3) + 14(1\ 2\ 3), \quad \beta = 6(1) + 2(2\ 3) - 7(1\ 3\ 2).$$

Compute $\alpha\beta$.

(c) Let $f : G \to G'$ be a homomorphism of monoids. Show there exists a unique R-algebra homomorphism $\phi : R[G] \to R[G']$ such that $\phi(g) = f(g)$ for all $g \in G$. (Recall an R-algebra is a ring A with an injective ring-homomorphism $\iota : R \hookrightarrow A$ such that $\iota(R) \subseteq Z(A)$; we usually drop ι and consider $R \subseteq A$. An R-algebra homomorphism $\phi : A \to A'$ is a ring homomorphism such that $\phi|_R = \mathrm{id}_R$.)

Date: Assigned Wednesday, 5 October 2016; due Friday, 7 October 2016.