MATH 101: ALGEBRA I
WORKSHEET, DAY #9

Problem JV9.A. Show that $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{C}$ and $\mathbb{C} \otimes_{\mathbb{C}} \mathbb{C}$ are both $\mathbb{R}$-vector spaces, and compute their dimensions. Are they isomorphic as $\mathbb{R}$-vector spaces?

Problem JV9.B. Let F be a field, let V be a finite-dimensional F-vector space, and let $T : V \times V \to F$ be a nondegenerate symmetric bilinear form. Let $W \subseteq V$ be a F-subspace. Define $W^{\perp} = \{v \in V : T(v, W) = 0\}$.

(a) Show that the map $V \to V^*$ by $v \mapsto T(v, -)$ maps $W^{\perp}$ isomorphically to $\operatorname{ann}(W)$.

(b) Deduce that $\dim V = \dim W + \dim W^{\perp}$.

(c) Suppose that $T|_{W \times W}$ is nondegenerate (accordingly, we say that W is a *nondegenerate* subspace under T). Show that $V = W \oplus W^{\perp}$. In this case, we say $W^{\perp}$ is the *orthogonal complement* of the nondegenerate subspace W.

(d) Define the *orthogonal projection* onto W. Then let $V = \mathbb{R}^3$ have the standard inner product and let

$$W = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 0\}$$

and compute its matrix with respect to the standard basis.

Date: Assigned Monday, 26 September 2016; due Tuesday, 27 September 2016.