ERRATA:
THE CANONICAL RING OF A STACKY CURVE

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This note gives errata for the article *The canonical ring of a stacky curve* [2]. The authors thank Eran Assaf, Sam Frengley, Evan O'Dorney, Tim Santens, and Sam Schiavone.

(1) Before (2.1.2): Should say "Applying this with $M = R_{\geq 1}$, we find that $\operatorname{Proj} R$ is a closed subscheme of the weighted projective space..."

(2) Section 4.3: replace "2-descent" with "repeated 2-descent".

(3) Lemma 5.6.10: first sentence should be "By hypothesis we have $\deg(K_{\mathscr{X}} + \Delta) < 0$, so there is a multiple m of $\operatorname{lcm}(1, e_1, \ldots, e_r)$ so that $m(K_{\mathscr{X}} + \Delta) = D$ is a nonstacky, very ample divisor on X."

(4) Example 5.7.1: the reference to GMNT in genus 1 should instead say "By GMNT in genus 1 (Lemma 3.1.4)".

(5) Examples 5.7.1, 5.7.3, and 5.7.4: should say $H^0(X, \lfloor (d/2)Q \rfloor)$, $H^0(X, \lfloor (3d/4)Q \rfloor)$, and $H^0(X, \lfloor d(1 - 1/e)Q \rfloor)$.

(6) Example 5.7.8: should say "Let $D' = \frac{1}{2}Q_1 + \frac{1}{2}Q_2$".

(7) Theorem 8.3.1(b): should say "Let $R' = k[x]/I' = k[x_1, \ldots, x_m]/I'$ be a presentation".

(8) Example 11.2.2 and Remark 11.2.4: both of these examples are wrong. Part (a) is not defined (the "projection morphism" is not defined), and the example in part (b) is not smooth. This makes Remark 11.2.4 nonsense in part (a) and irrelevant in part (b); but it does not impact the rest of the paper. In fact, all tame relative stacky curves are twisted! See Santens [1, Lemma 2.1].

(9) Table (IIa), $g = 1$, $\delta \geq 4$: off by one error in the last column, should have $P(I; t) = \frac{\delta(\delta - 3)}{2}t^2$.

References

[1] Tim Santens, *The Brauer–Manin obstruction for stacky curves*, 2022, `arXiv:2210.17184`.

[2] John Voight and David Zureick-Brown, *The canonical ring of a stacky curve*, Mem. Amer. Math. Soc. **277** (2022), no. 1362.