Which numbers have square roots?

Reed College Senior Thesis Presentation

Splitting Probabilities of p-adic Polynomials

Division of Mathematics and Natural Sciences

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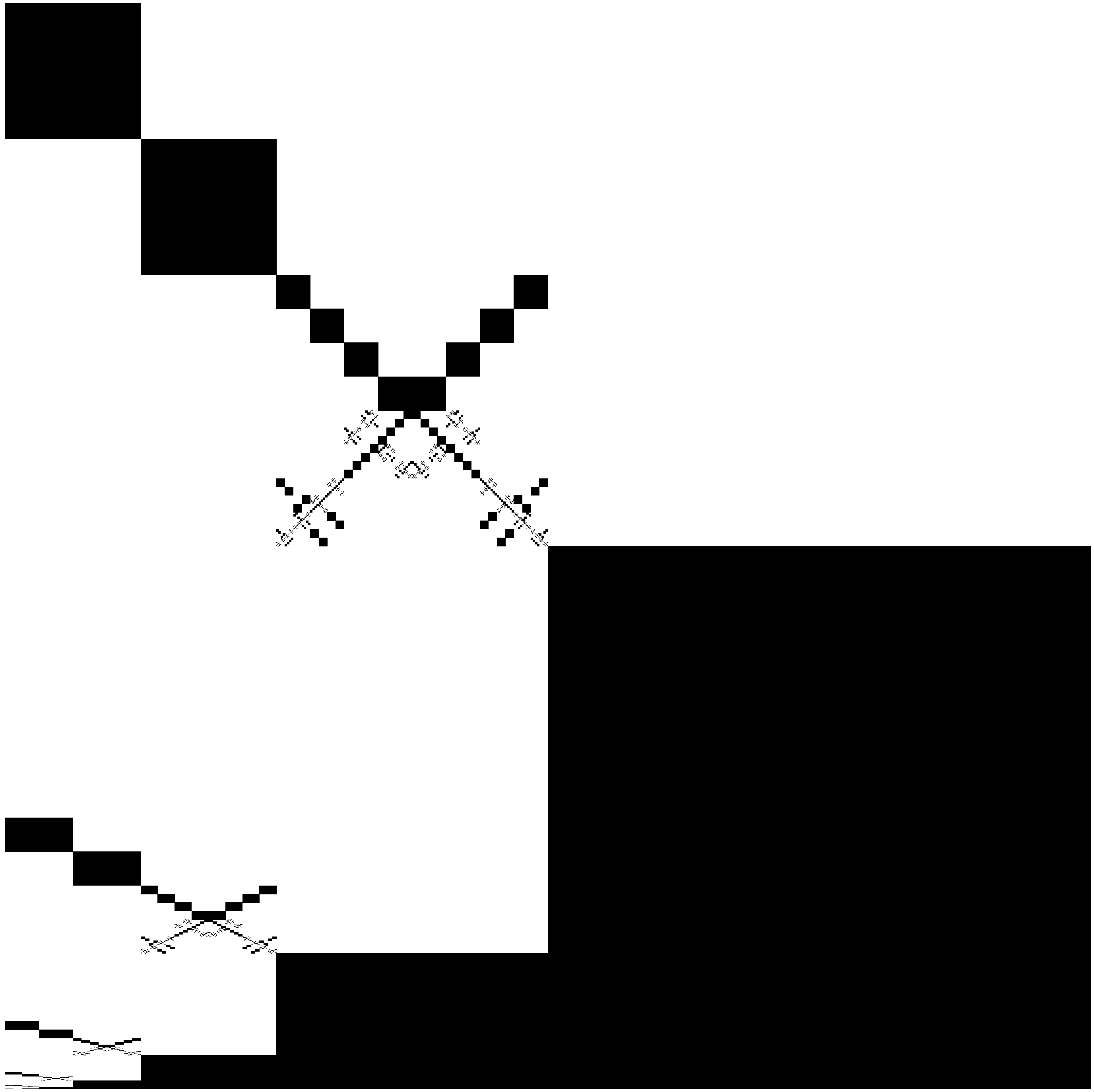

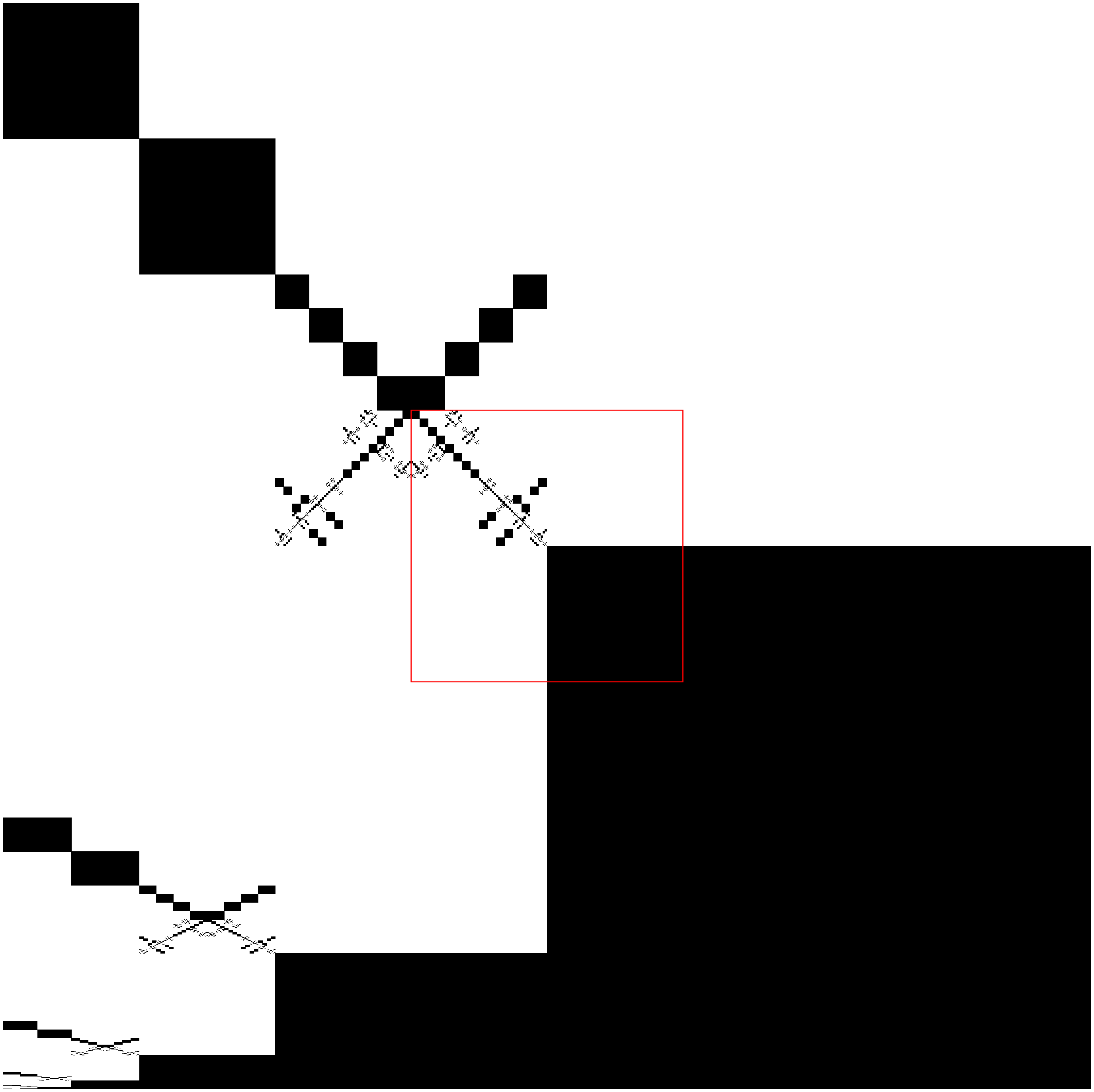

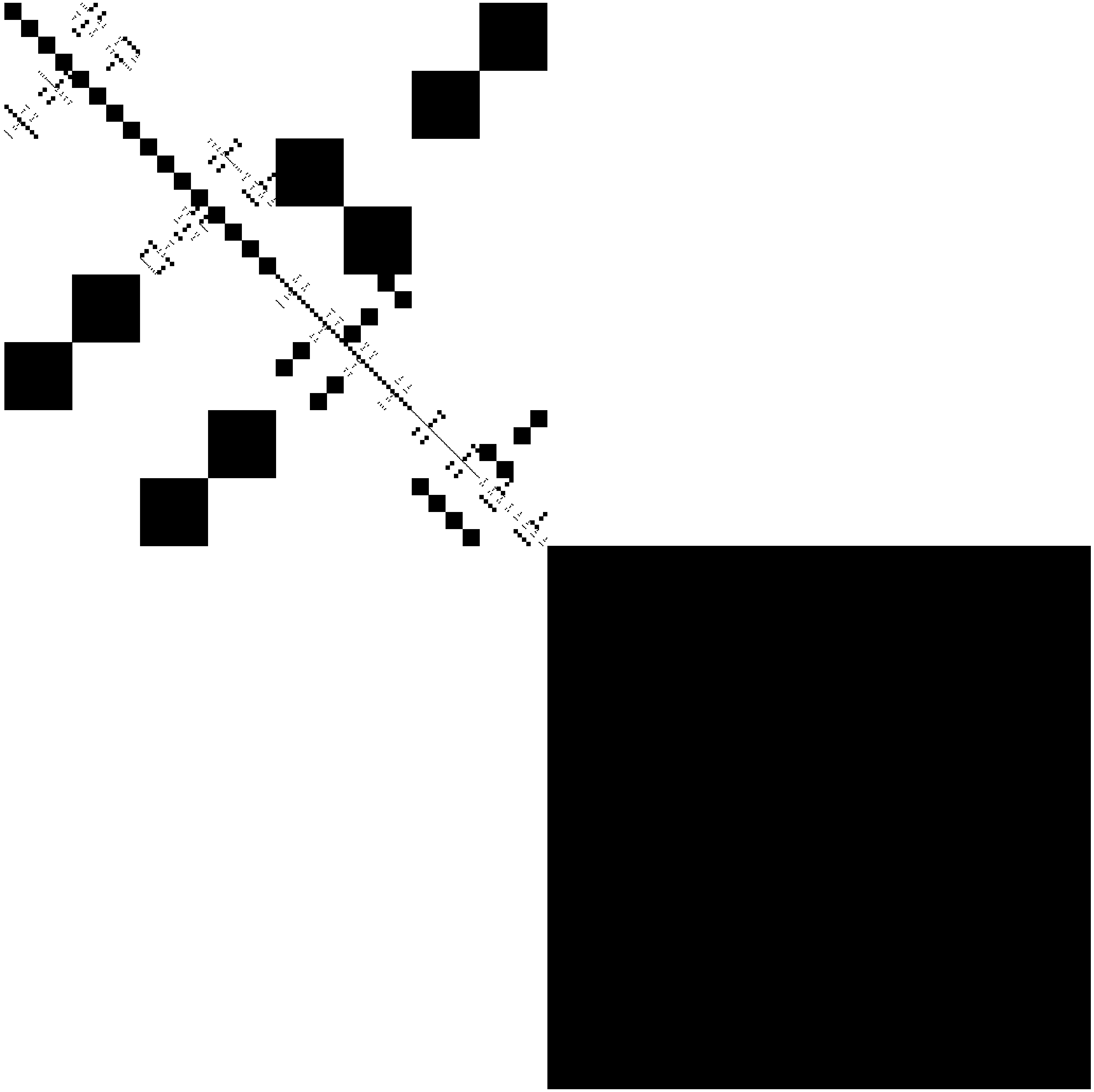

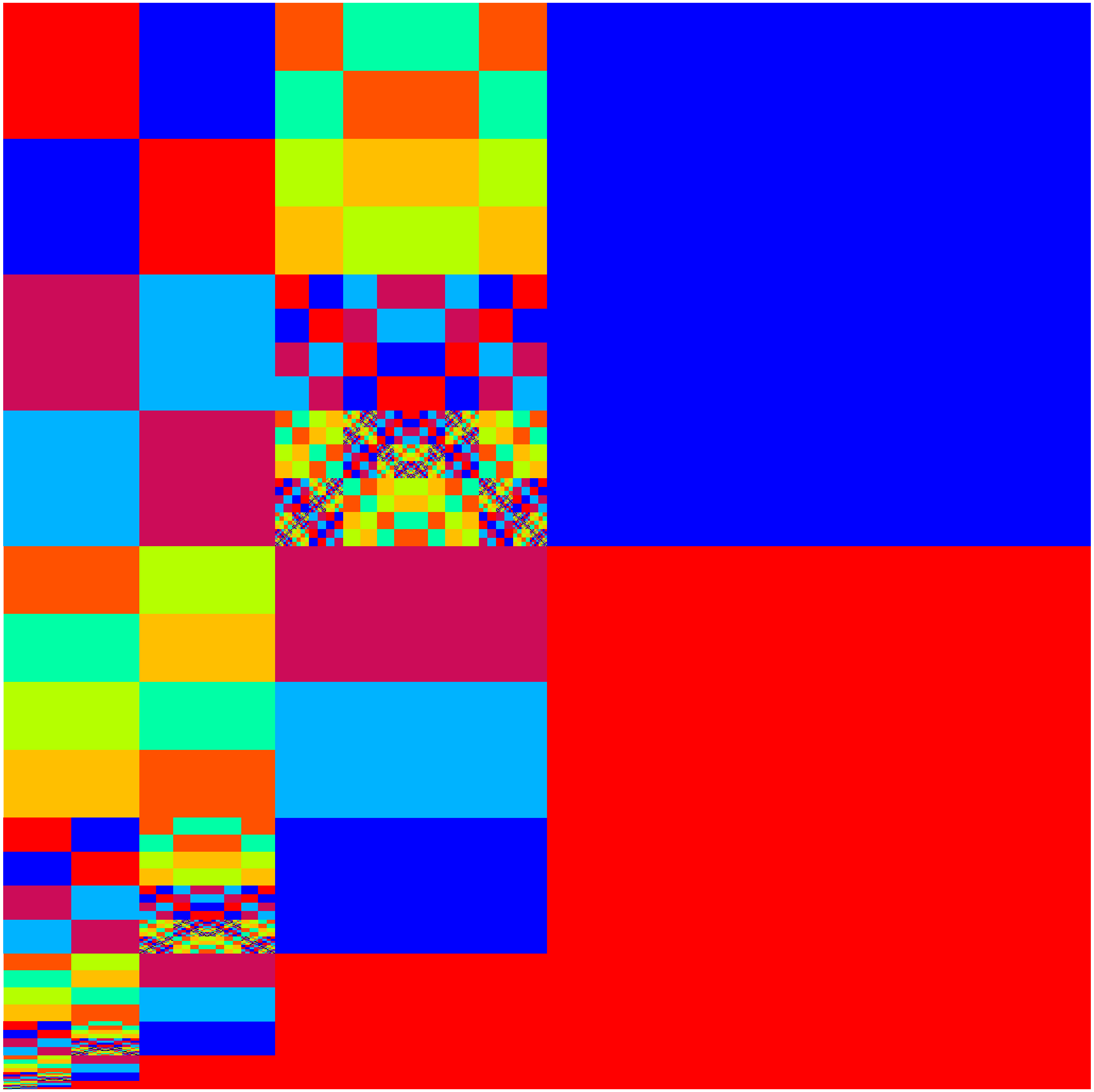

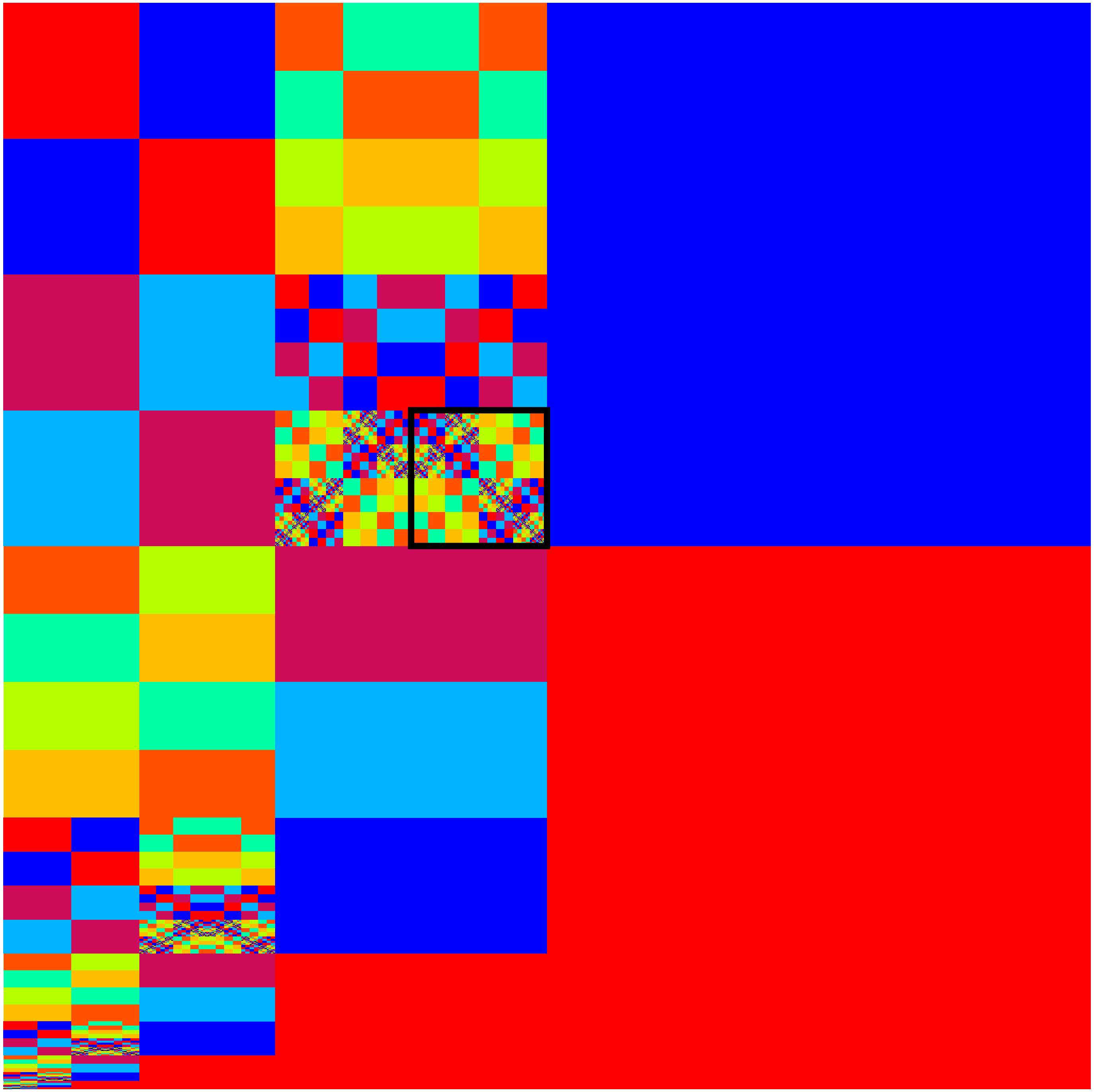

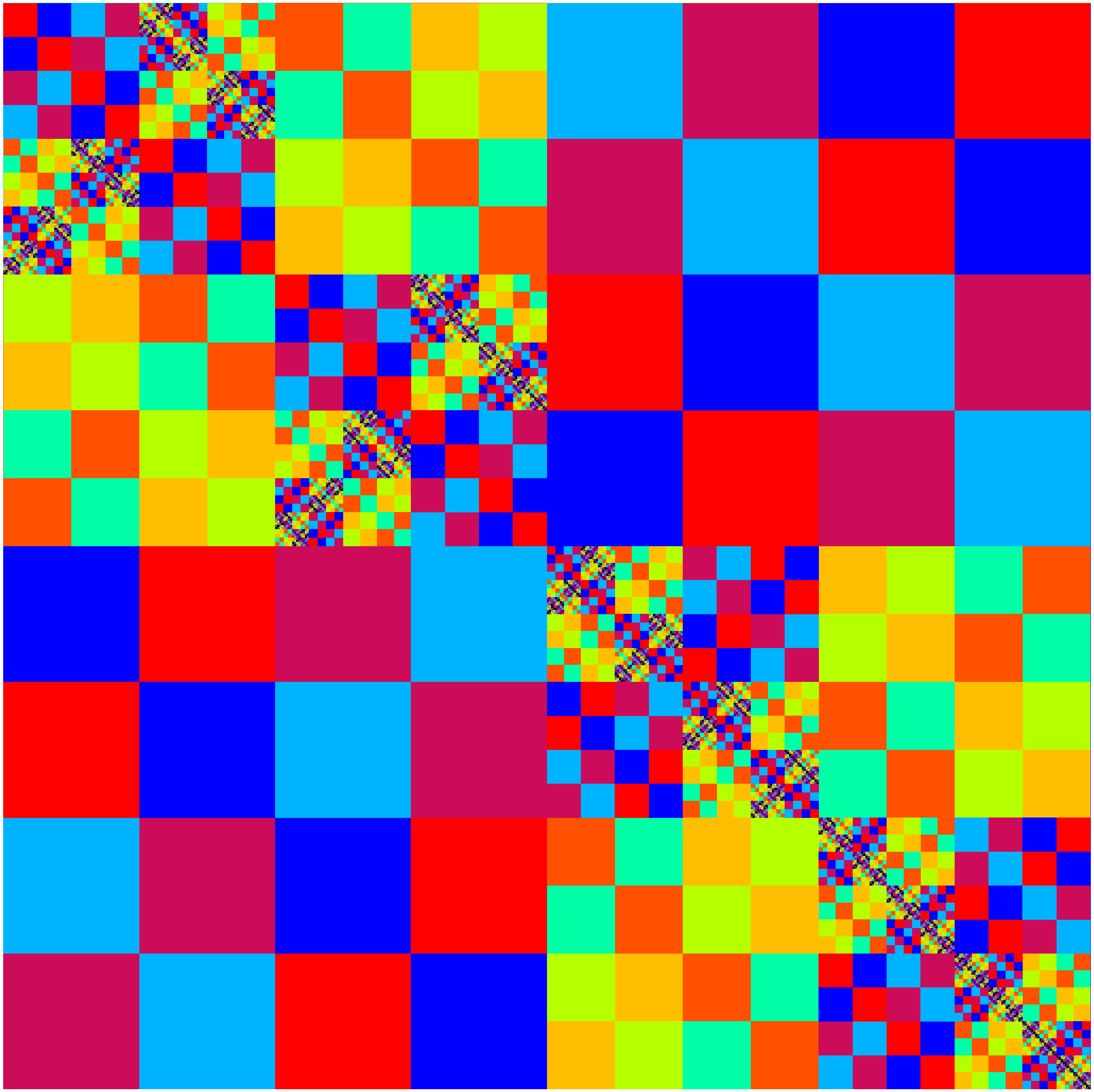

Which numbers have square roots?

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It depends on the rules.

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For whole numbers

$$0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9, \ldots$$

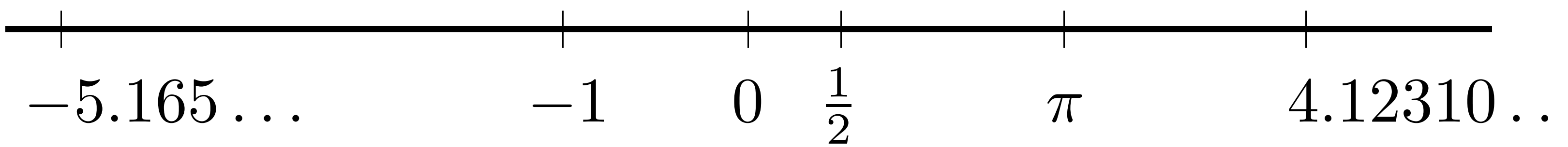
only *perfect squares* have square roots

$$0,\ 1,\ 4,\ 9,\ 16,\ 25,\ 36,\ 49, \ldots$$

Which numbers have square roots?

It depends on the rules.

For real numbers on the number line



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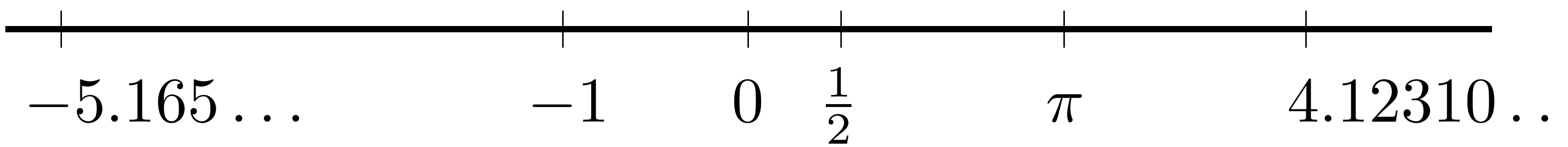
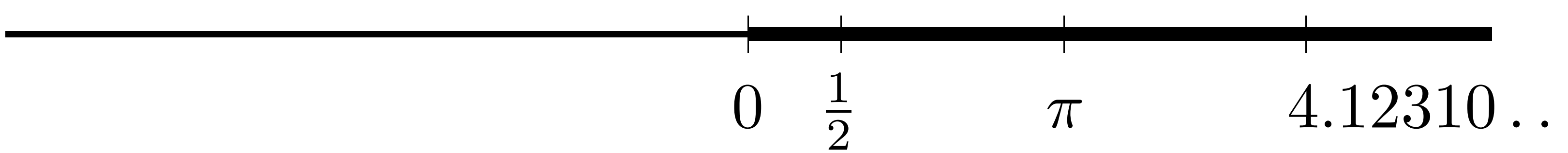


only *non-negative* numbers have square roots



in particular, only -1 doesn't have a square root.

$\sqrt{17} = 4.12301\ldots$ is irrational!

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[Theodorus] was proving to us a certain thing about square roots, I mean the side (i.e. root) of a square of three square units and of five square units, that these roots are not commensurable in length with the unit length, and he went on in this way, taking all the separate cases up to the root of seventeen square units, at which point, for some reason, he stopped.

-Plato, *Theaetetus*

$$\sqrt{17} = 4.12301\ldots \text{ is irrational!}$$

But we can get close!

$\sqrt{17} = 4.12301\ldots$ is irrational!

But we can get close!

x	$x^2 - 17$				
4.1	$(4.1)^2 - 17$	$=$	$-.19$	$=$	-2×10^{-1}
4.12	$(4.12)^2 - 17$	$=$	$-.026$	$=$	-3×10^{-2}
4.123	$(4.123)^2 - 17$	$=$	$-.00087\ldots$	$=$	-9×10^{-4}
$4.1\ldots$	$(4.1\ldots)^2 - 17$	$=$	ϵ	$=$	$u \times 10^{-n}$

$$4,\ 4.1,\ 4.12,\ 4.123,\ 4,1230\ldots \ \rightarrow\ \sqrt{17}$$

$\sqrt{17} = 4.12301\ldots$ is irrational!

A silly attempt, but why must we choose 10 over 2?

x	$x^2 - 17$				
3	$3^2 - 17$	$=$	-8	$=$	-1×2^3
7	$7^2 - 17$	$=$	32	$=$	1×2^5
23	$23^2 - 17$	$=$	512	$=$	1×2^9
279	$279^2 - 17$	$=$	77824	$=$	19×2^{12}
$?$	$?^2 - 17$	$=$	λ	$=$	$u \times 2^n$

$$3,\ 7,\ 23,\ 279, \ldots \ \rightarrow \ ?$$

$$
\begin{array}{rcl}
3 &=& 1 + 1\cdot 2 \\
7 &=& 1 + 1\cdot 2 + 1\cdot 2^2 \\
23 &=& 1 + 1\cdot 2 + 1\cdot 2^2 + 0\cdot 2^3 + 1\cdot 2^4 \\
279 &=& 1 + 1\cdot 2 + 1\cdot 2^2 + 0\cdot 2^3 + 1\cdot 2^4 + 1\cdot 2^8 \\
\vdots & &
\end{array}
$$

$$
\begin{array}{l}
11 \\
111 \\
11101 \\
111010001 \\
111010001001 \\
\vdots \\
\downarrow \\
\sqrt{17}
\end{array}
$$

Which p-adic numbers have square roots?

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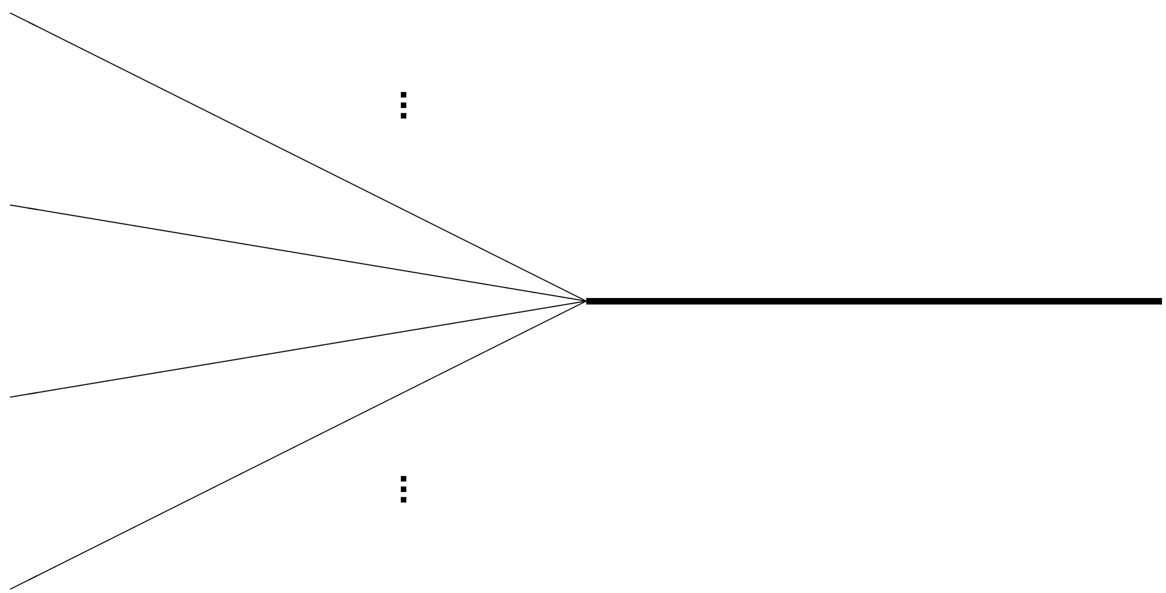
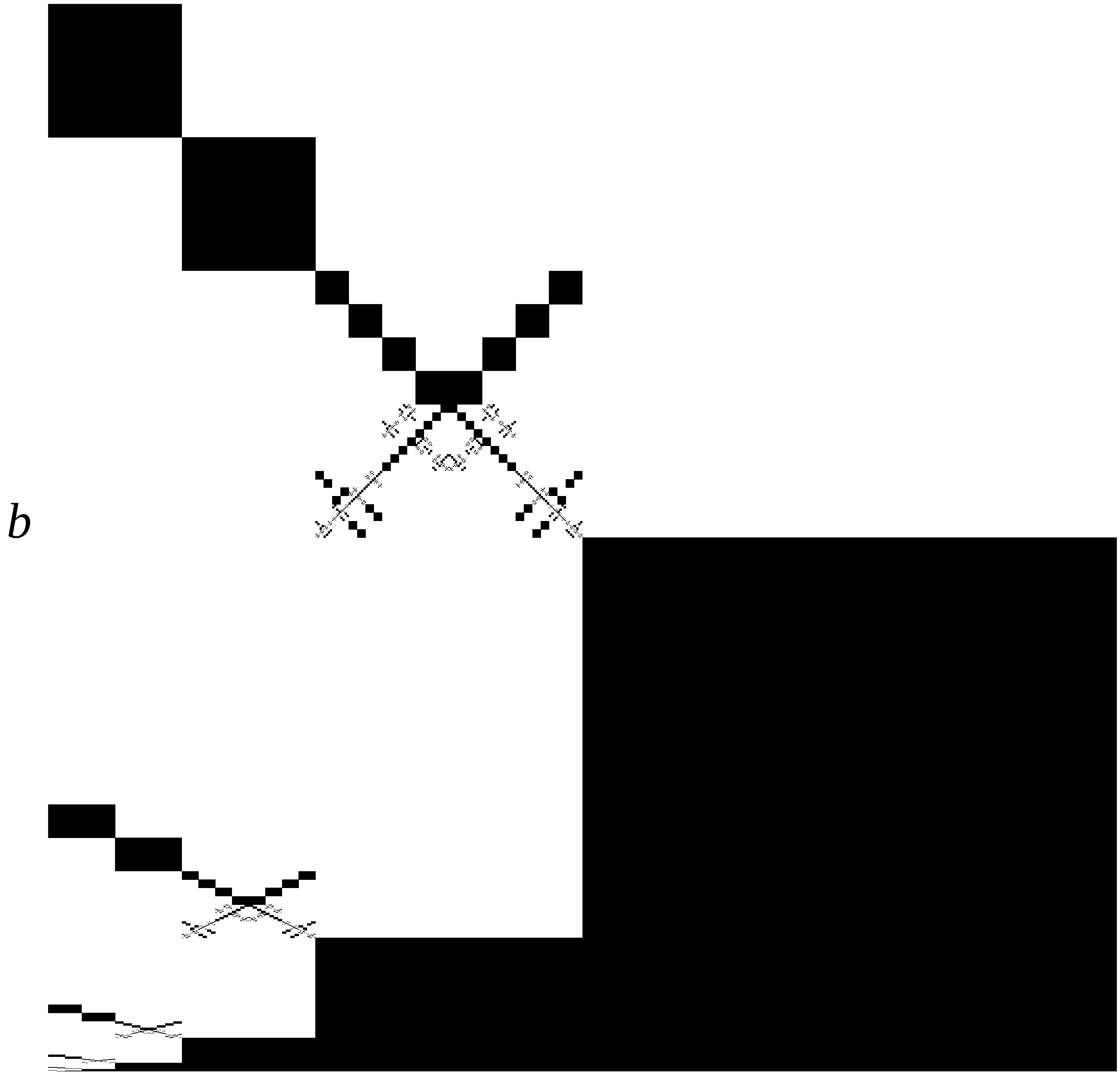
It depends on p.

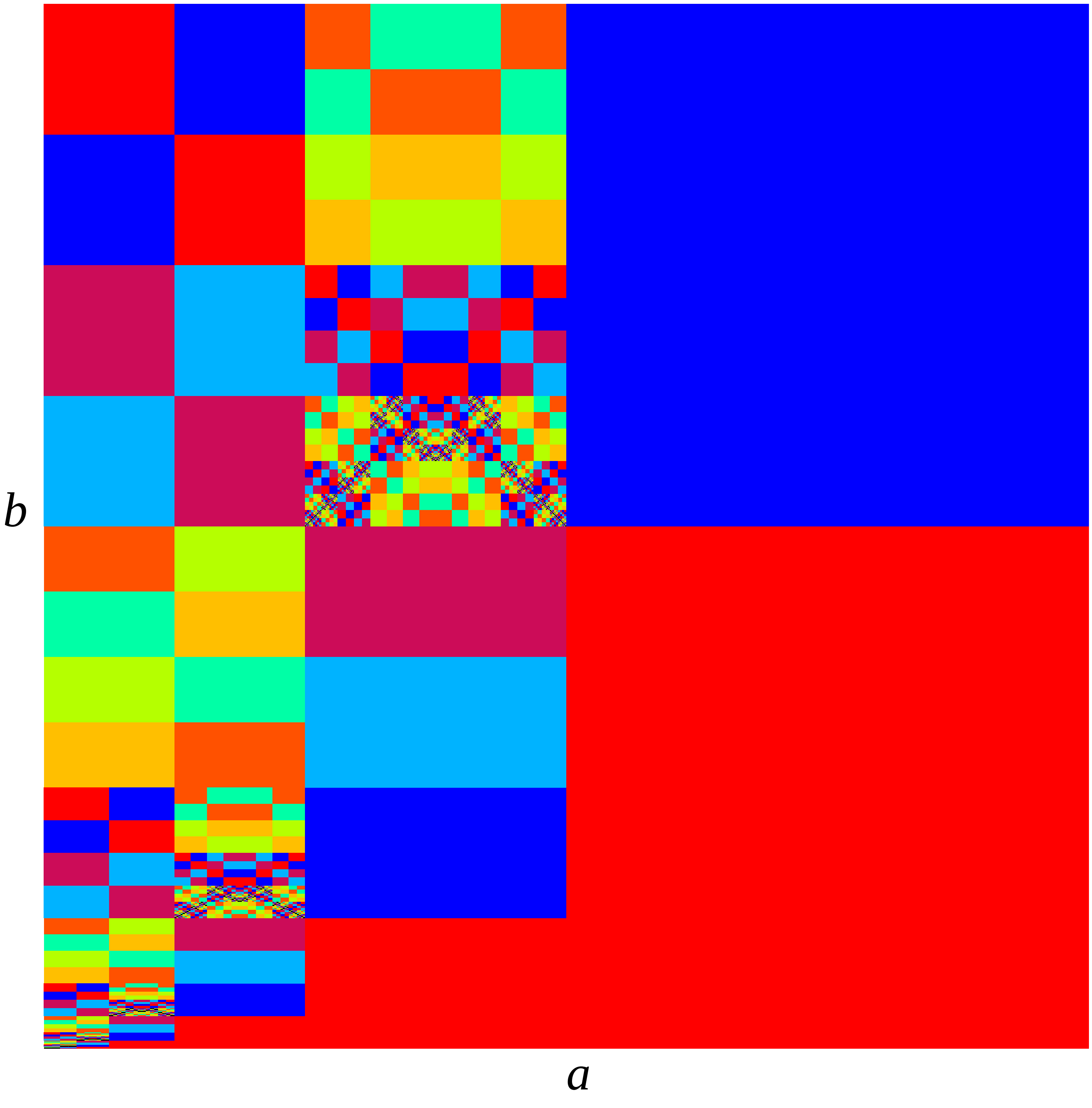
For general p-adic numbers,



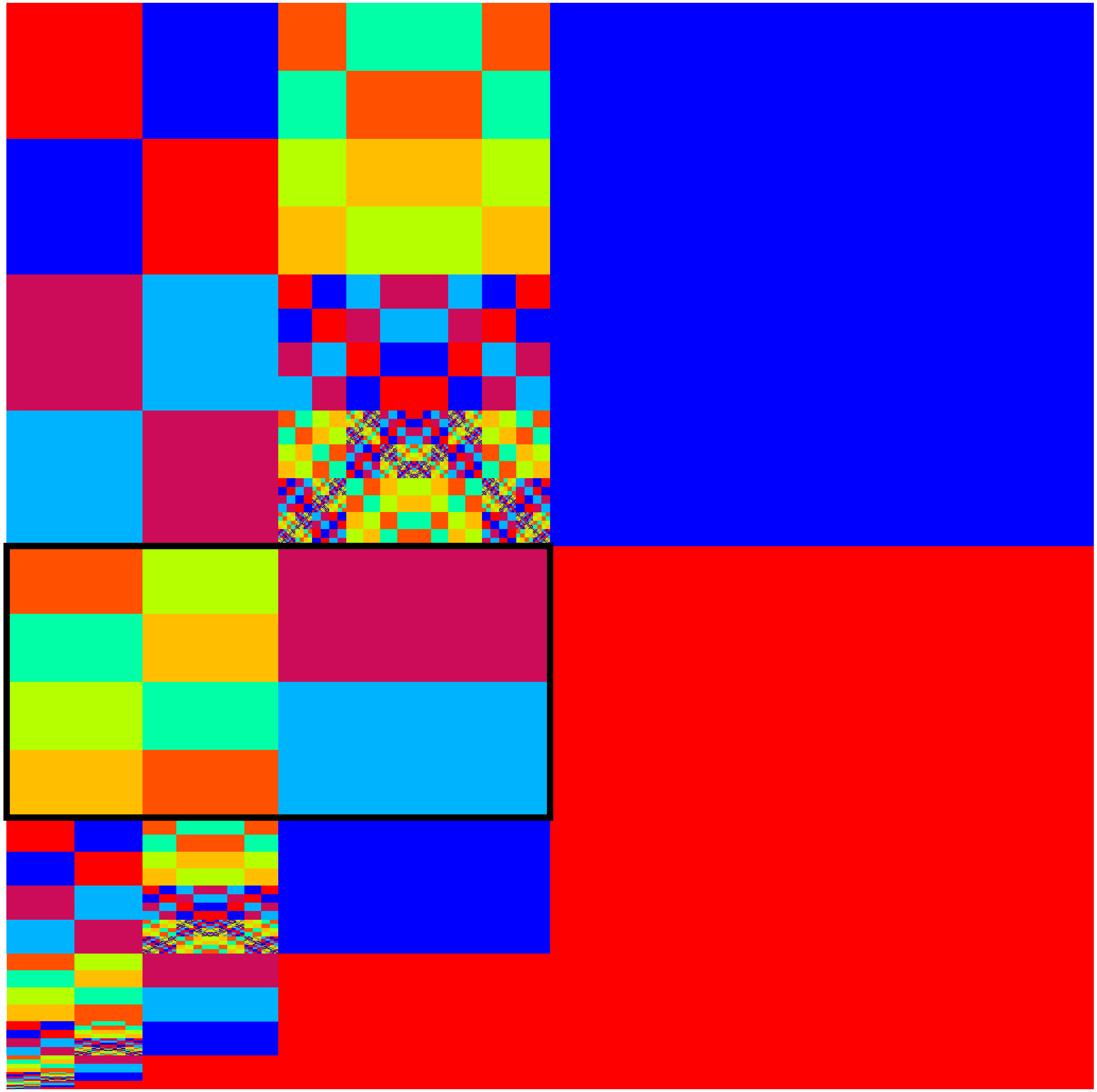
there are multiple square roots missing.



x^2+ax+b has a root? a,b are 2-adic numbers.



x^2+ax+b has which root? a,b are 2-adic numbers.



J. P. Serre, 1968