Pick's Theorem

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Pathways to Science Café
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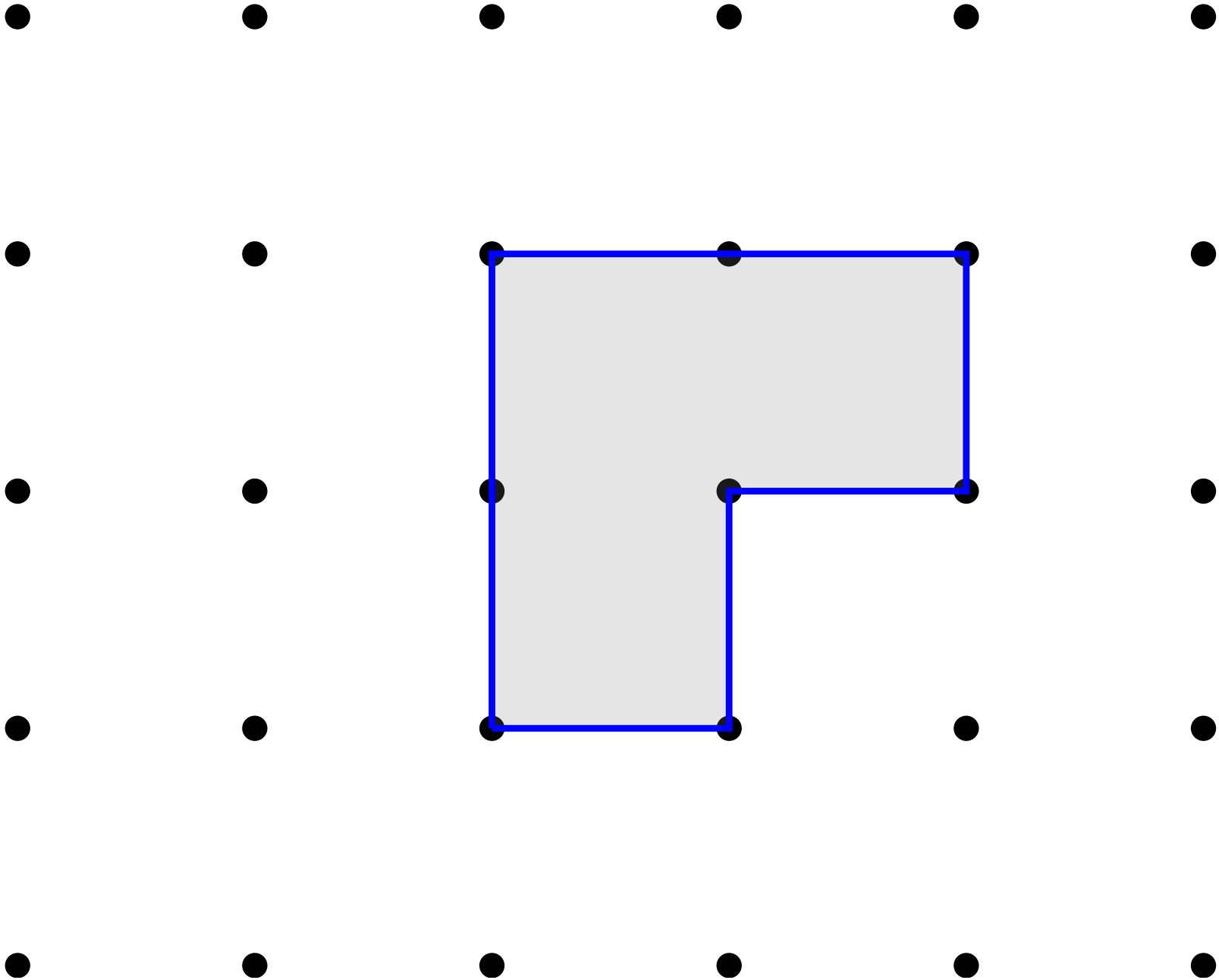
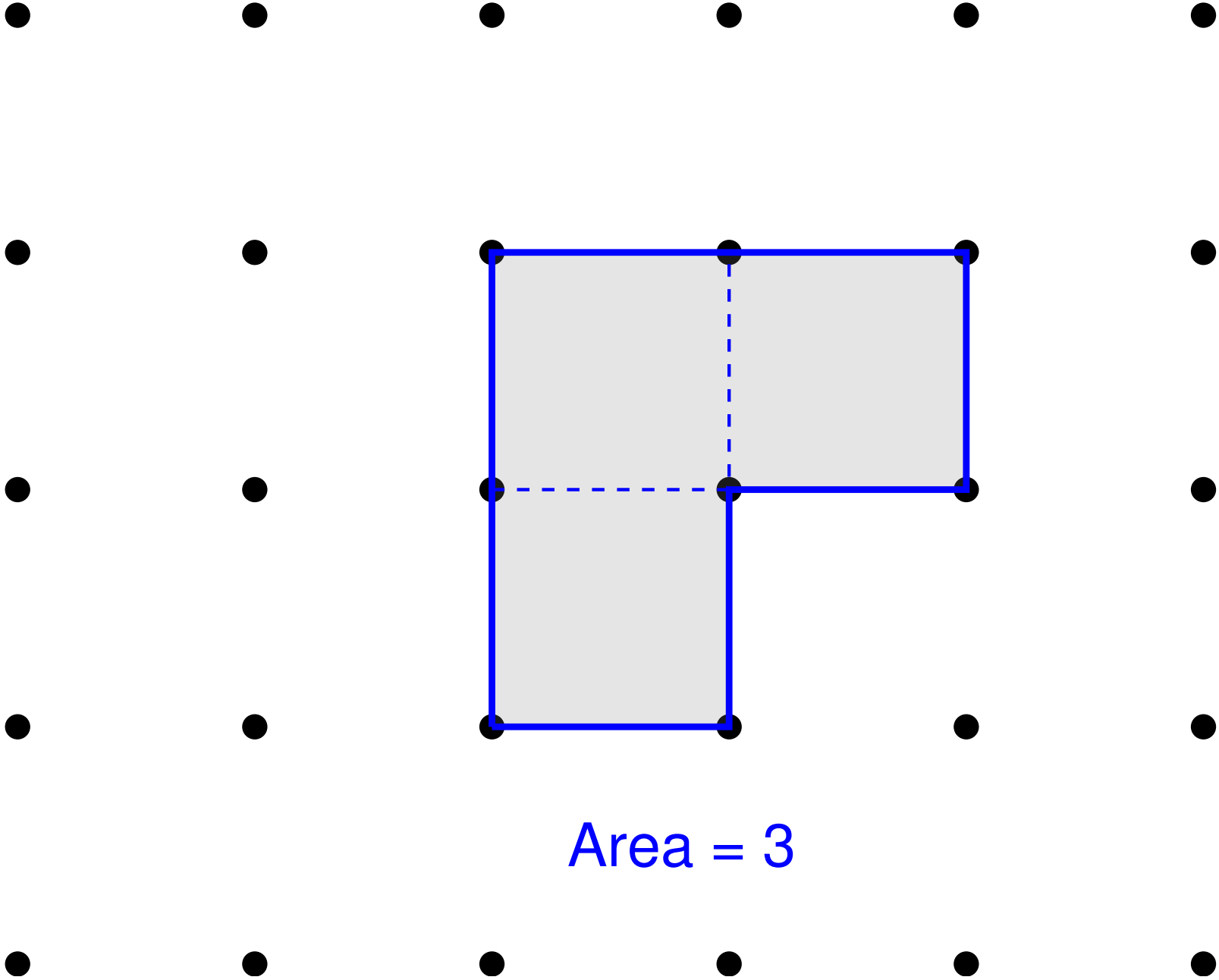
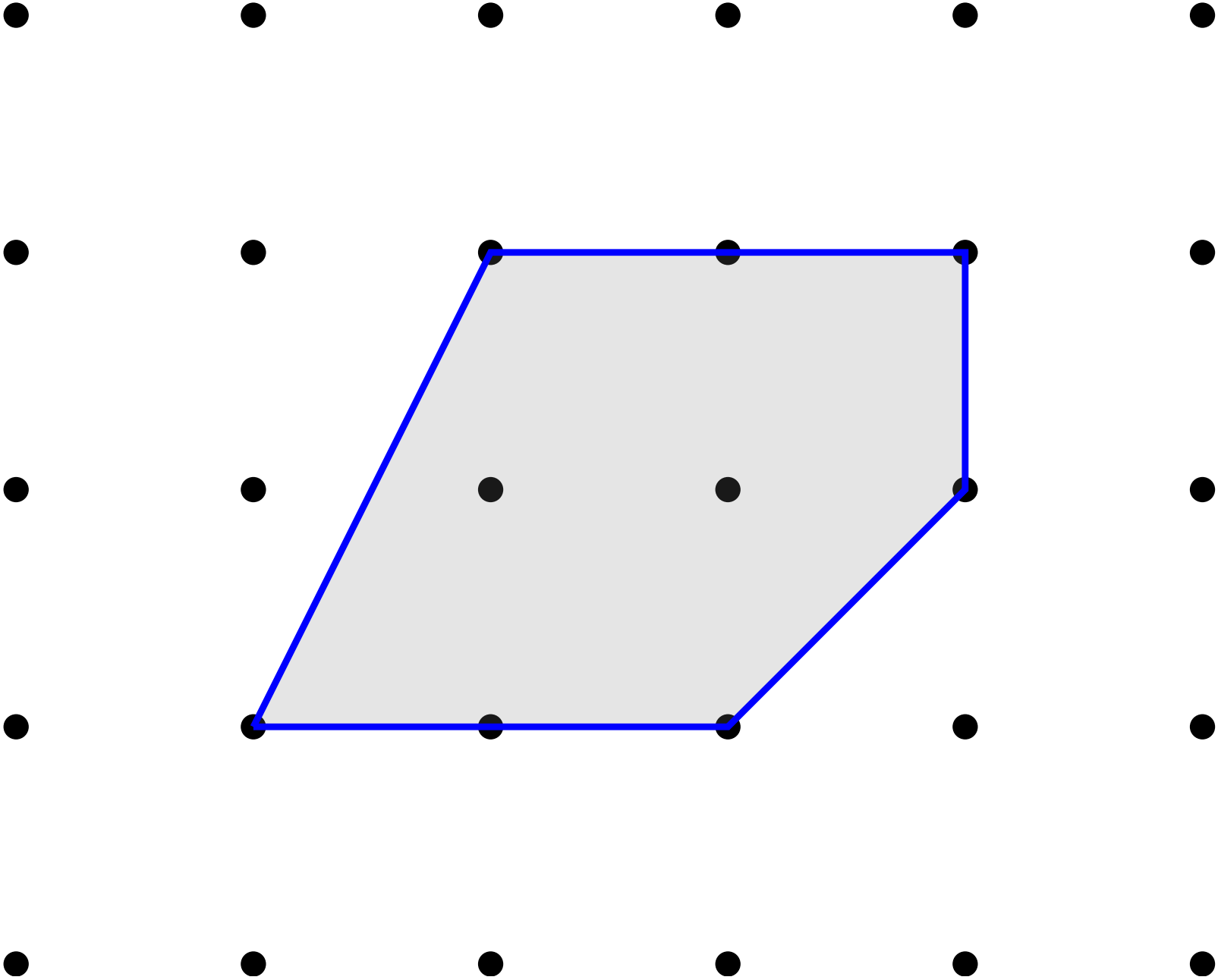
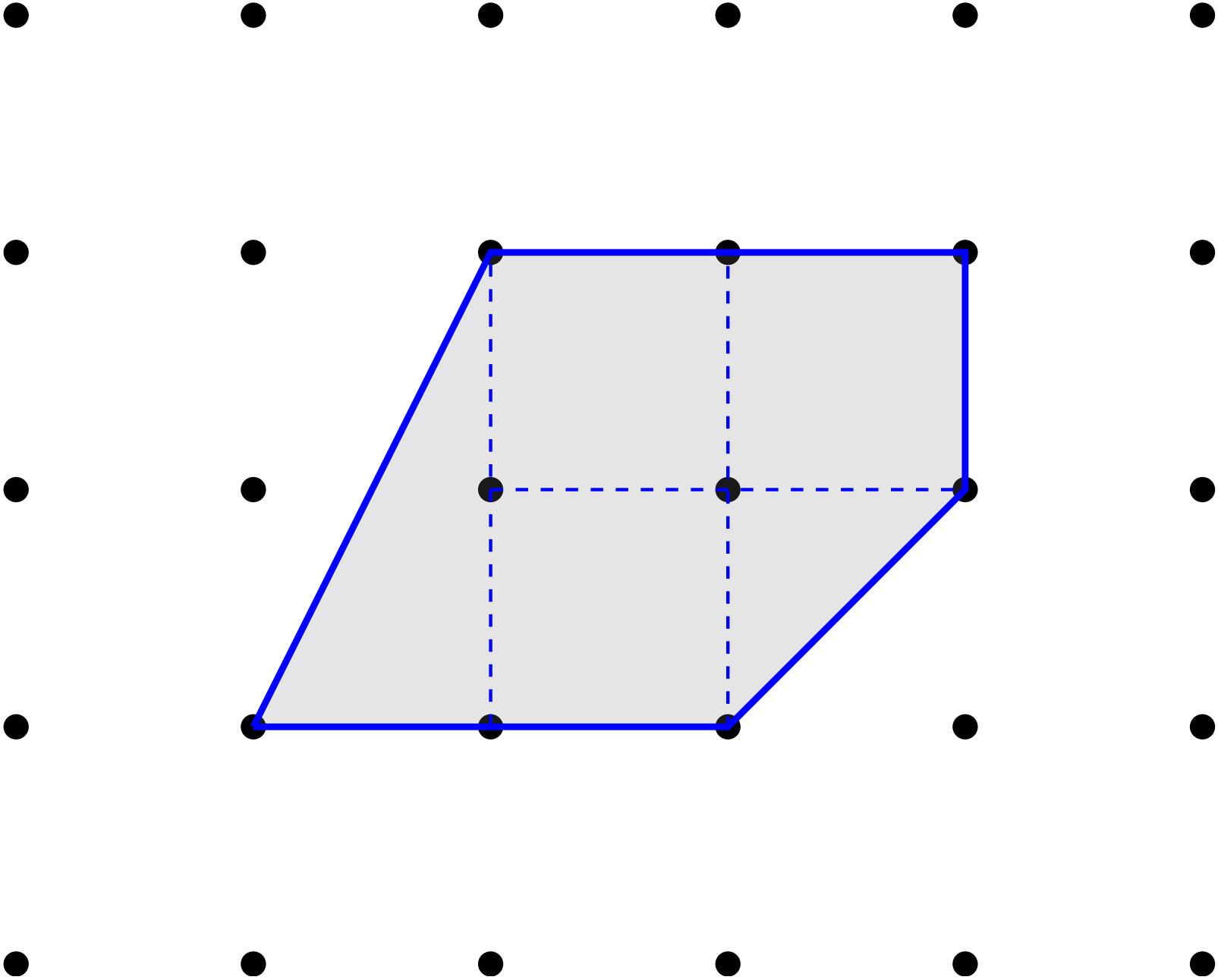
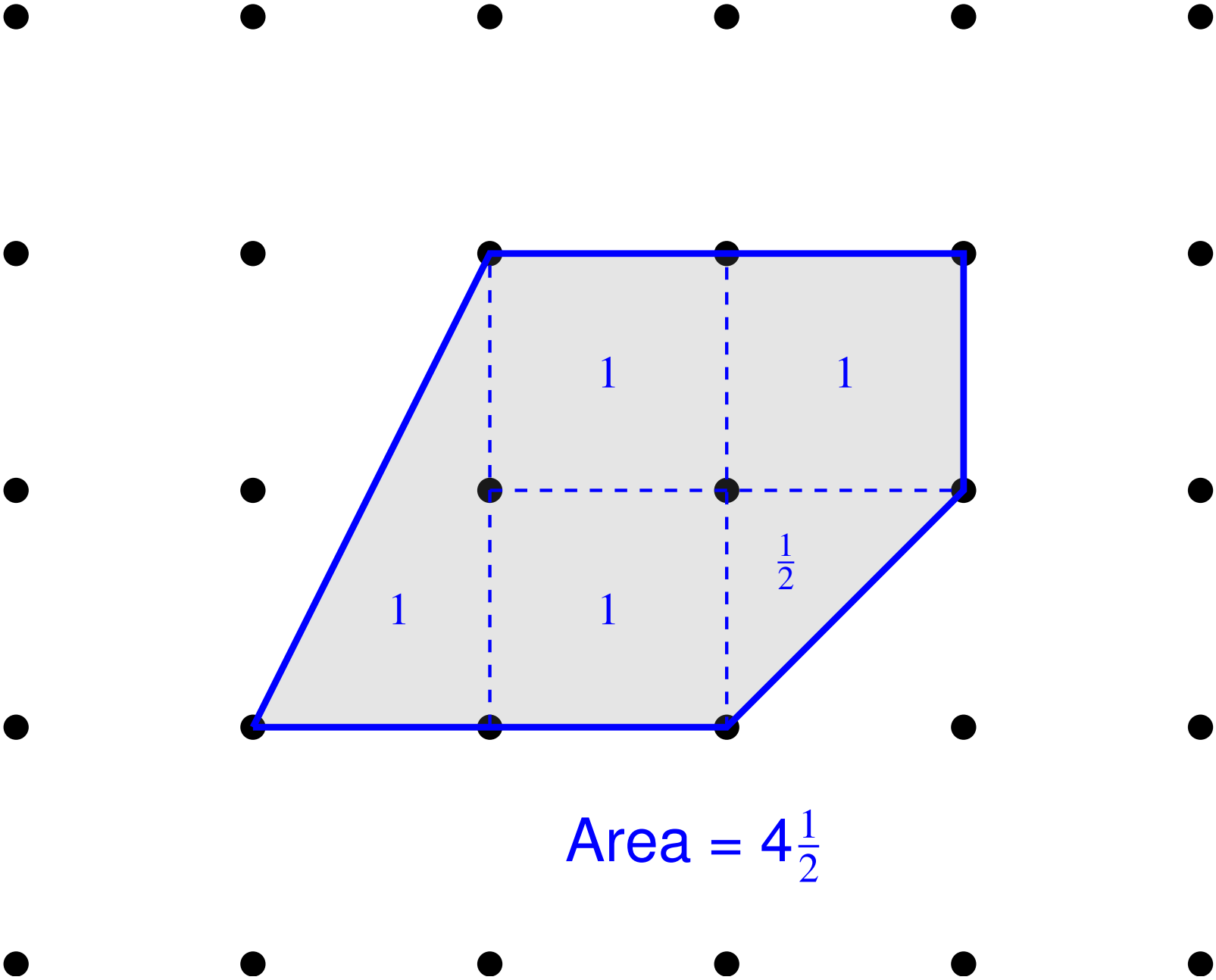


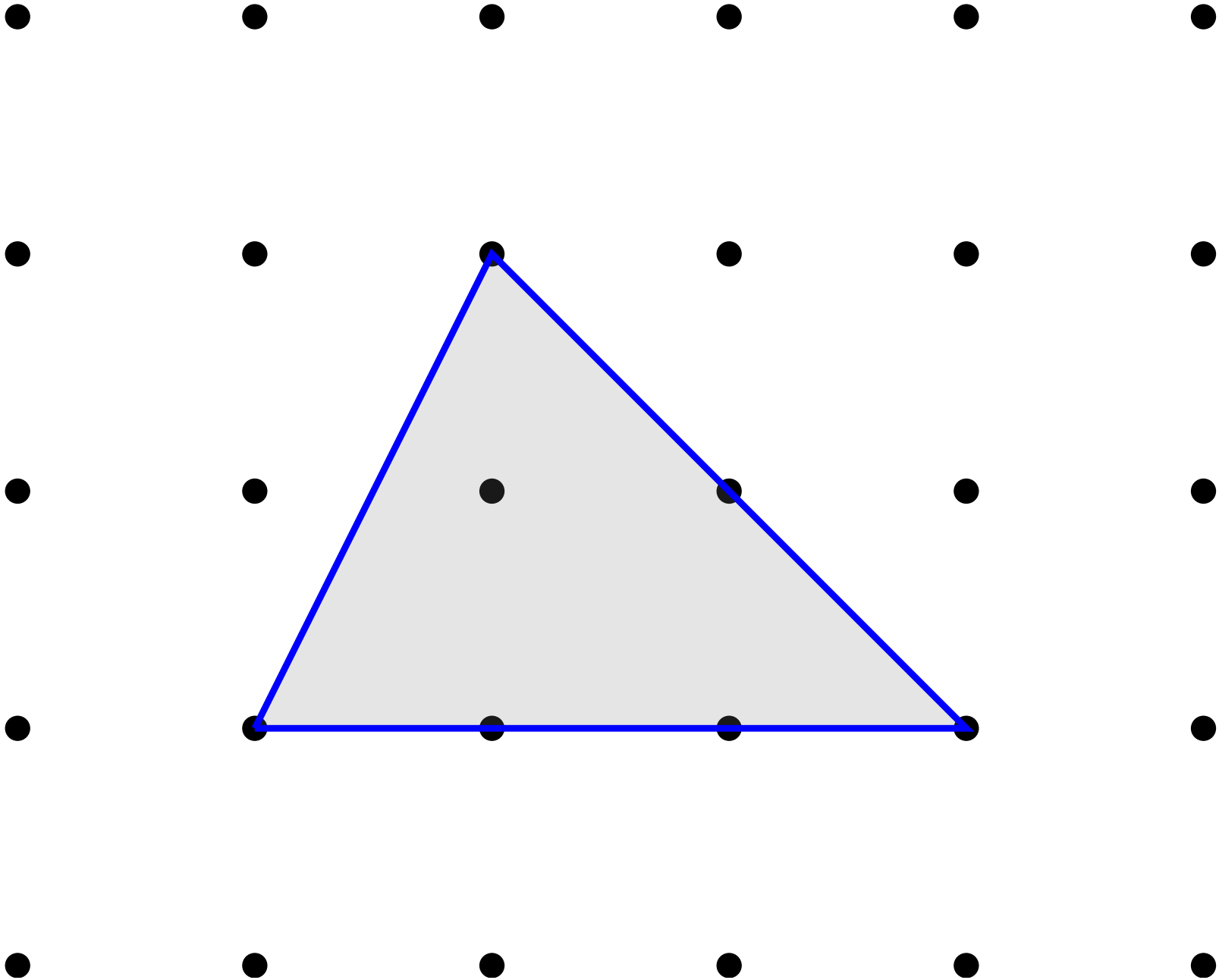

Area = 3

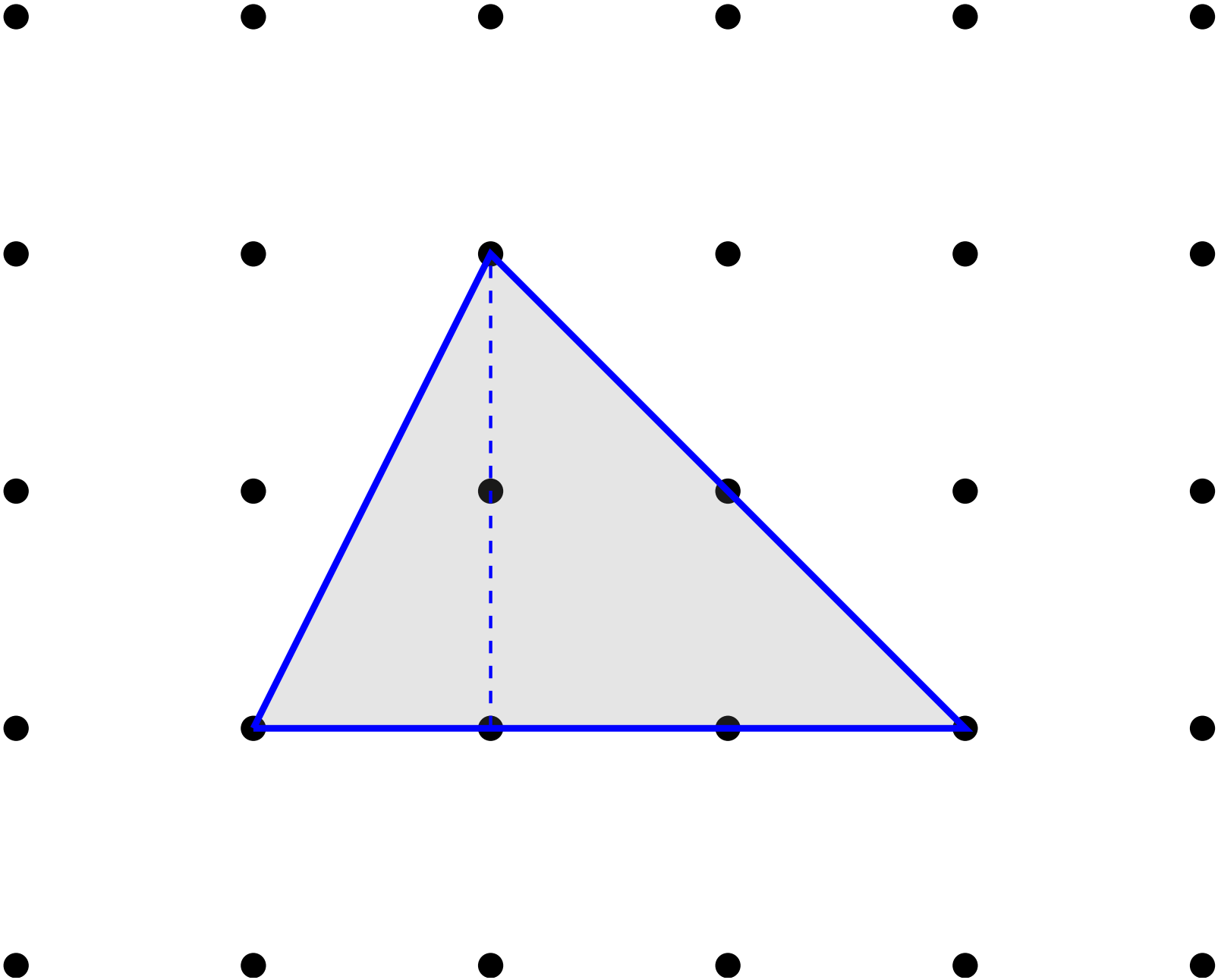


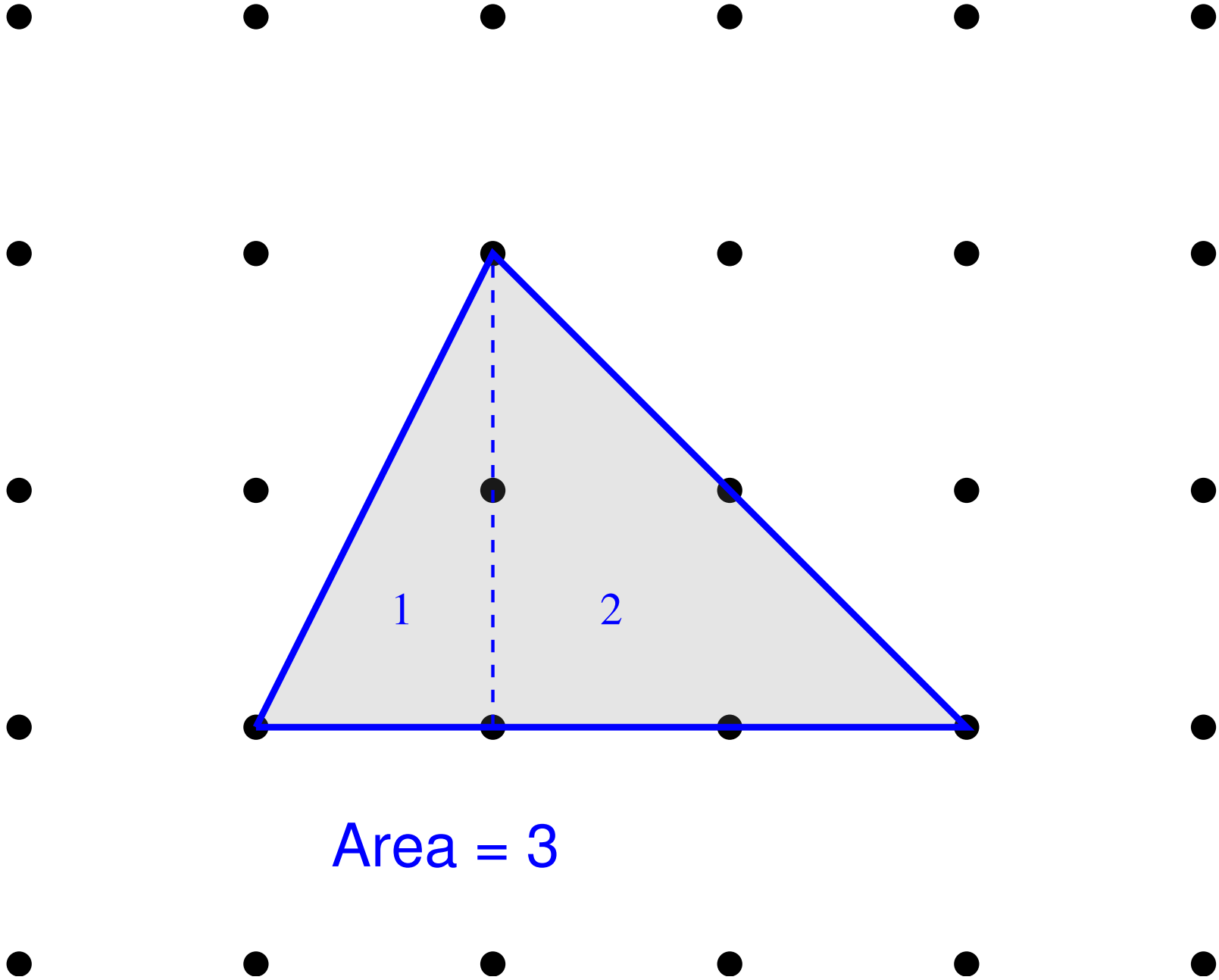


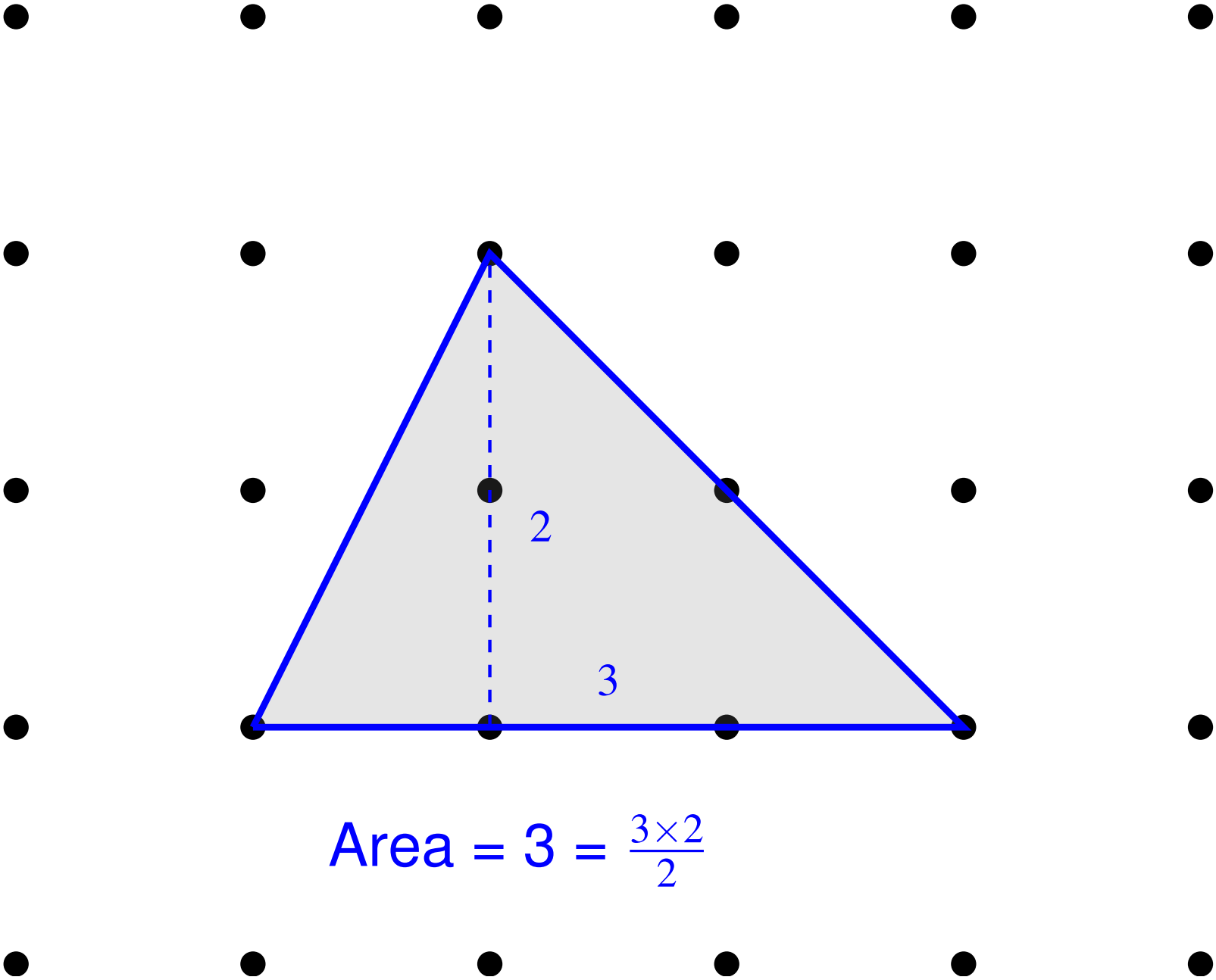


Area = $4\frac{1}{2}$

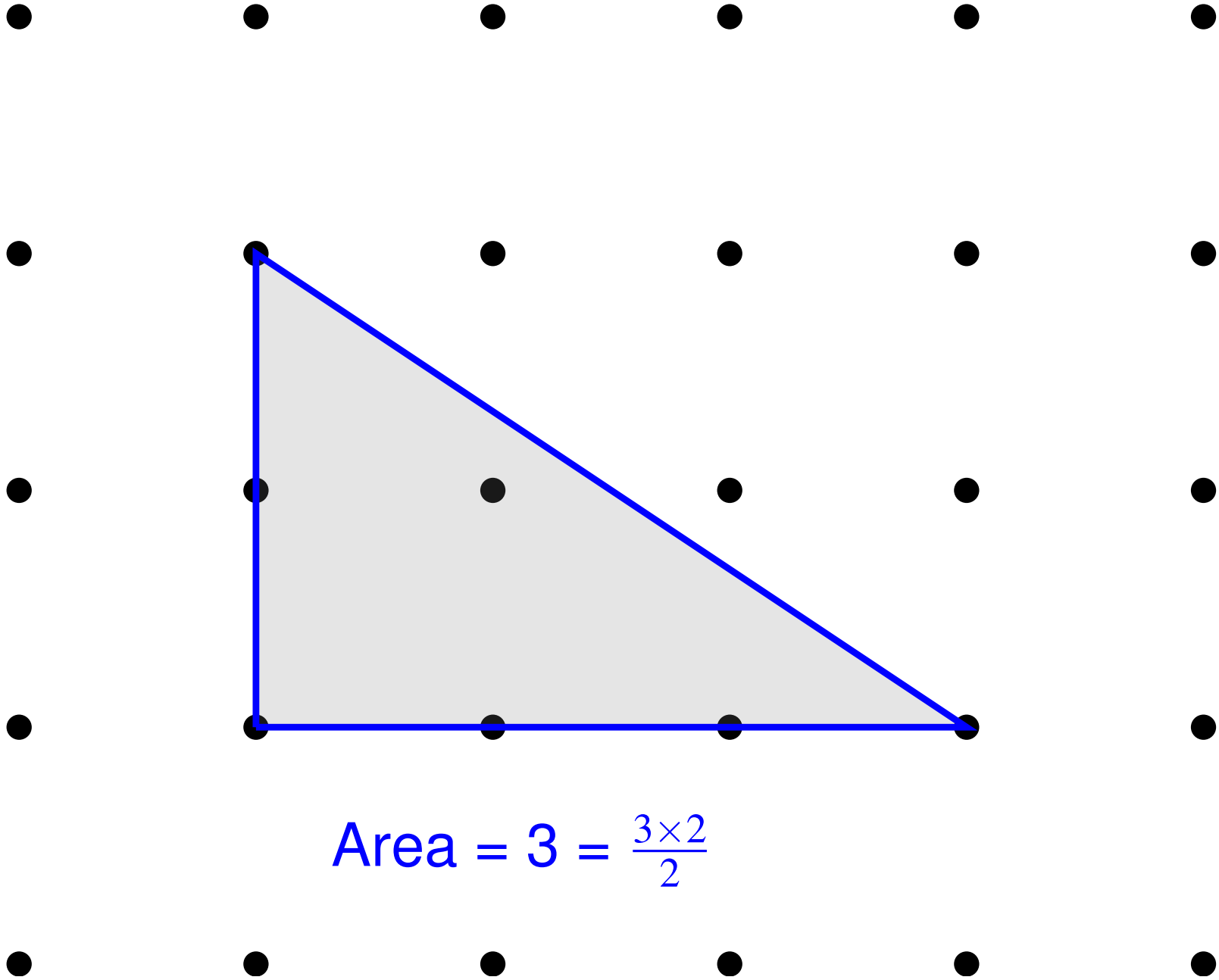




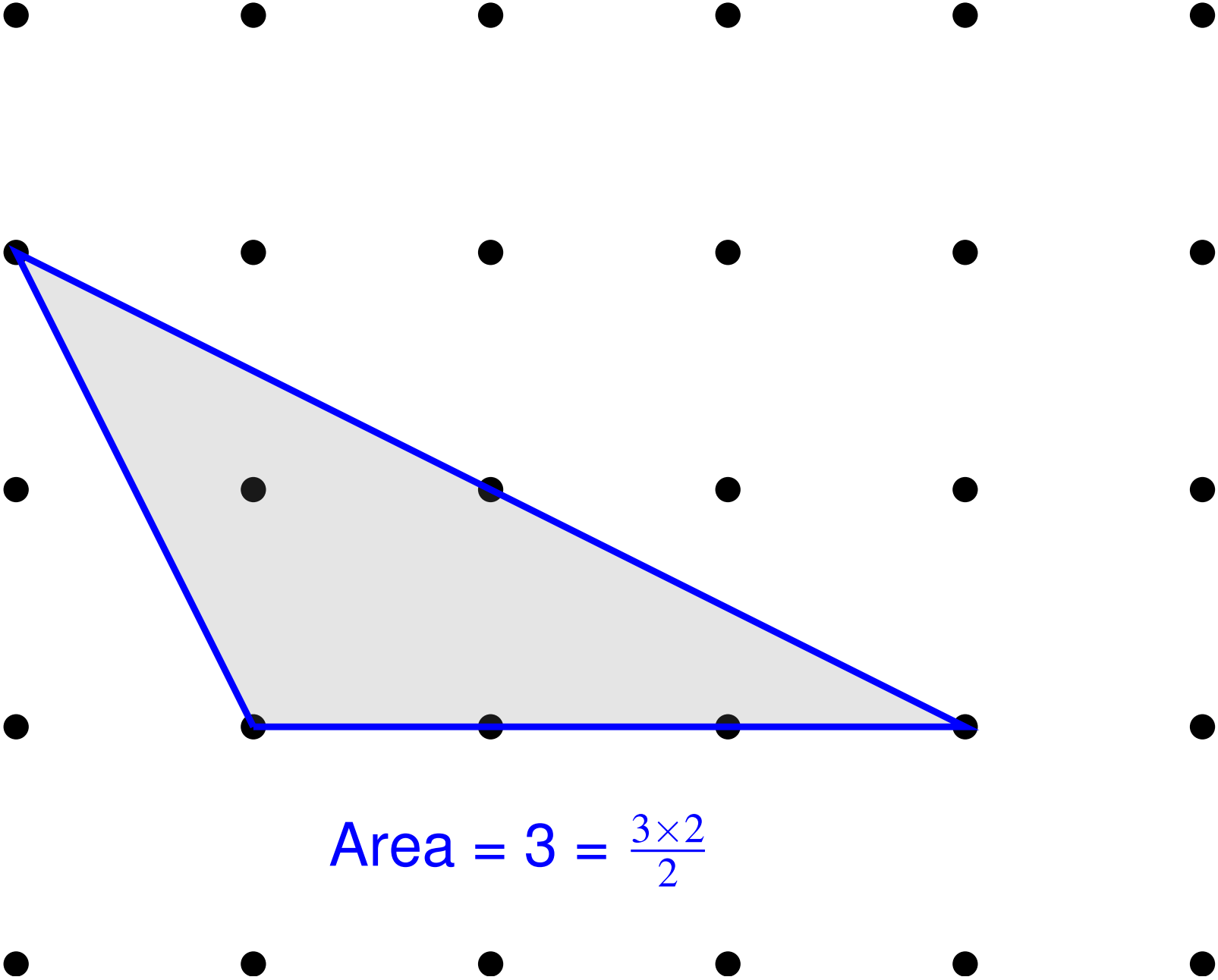

1
2
Area = 3



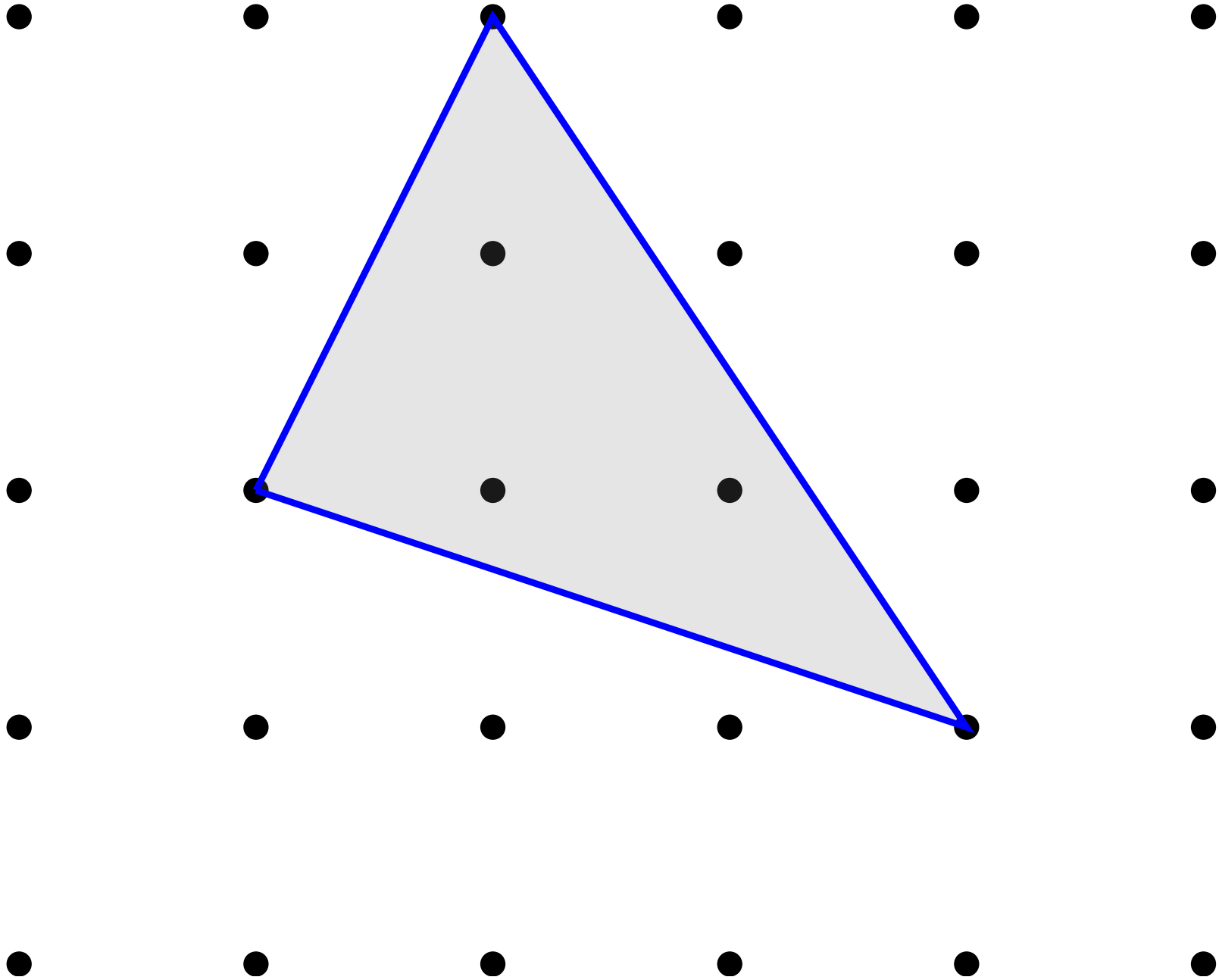
Area = 3 = $\frac{3 \times 2}{2}$

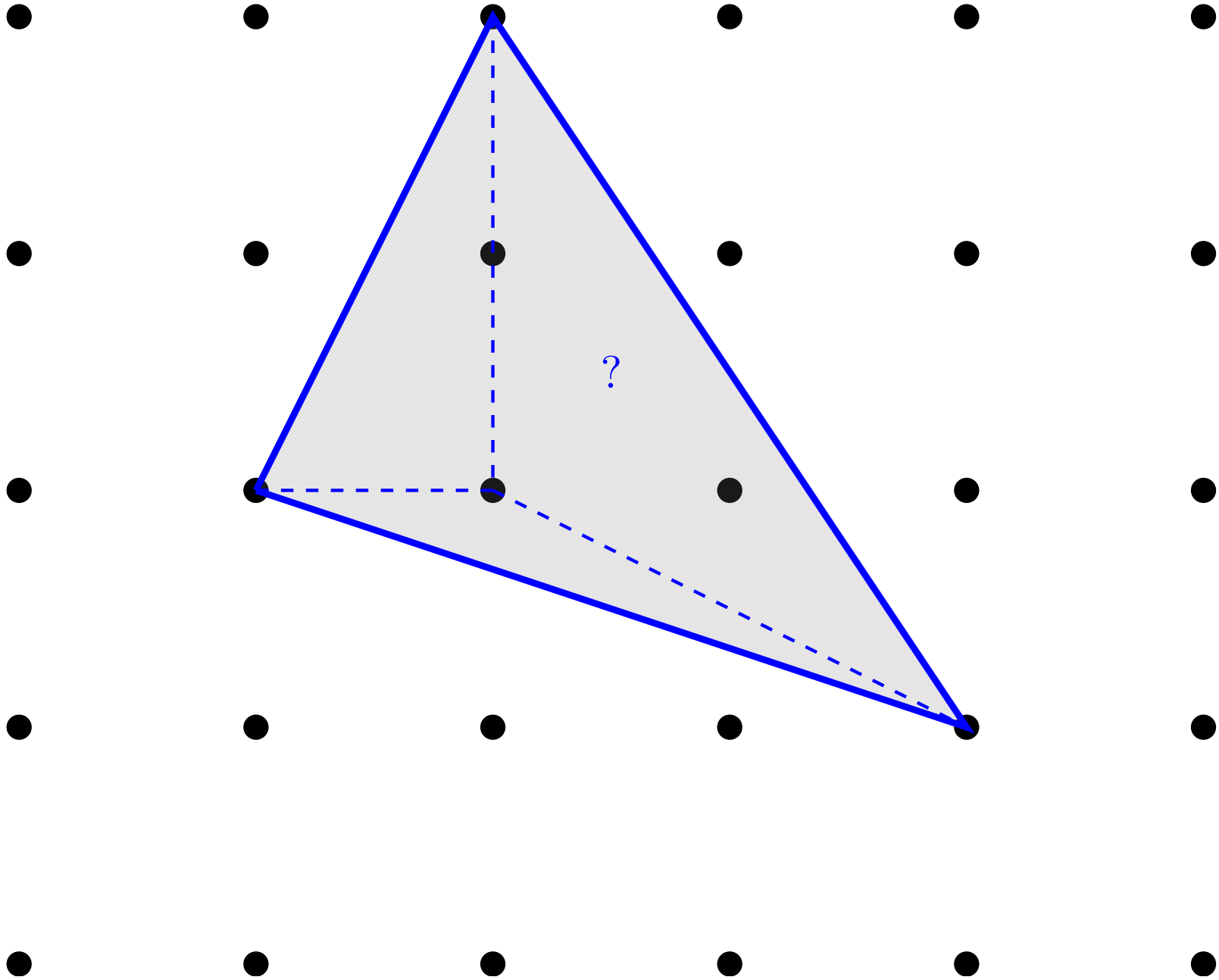


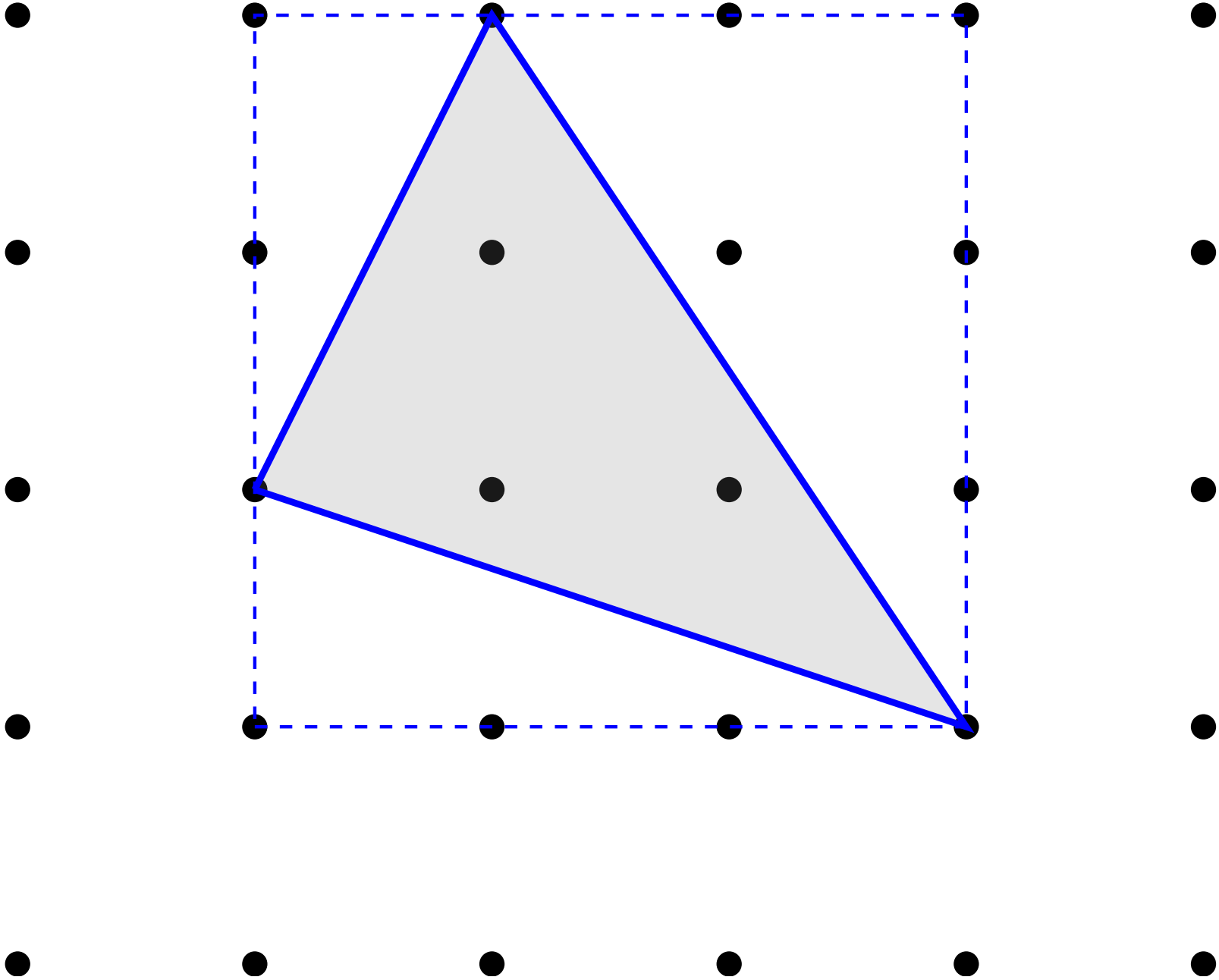
Area = 3 = $\frac{3 \times 2}{2}$

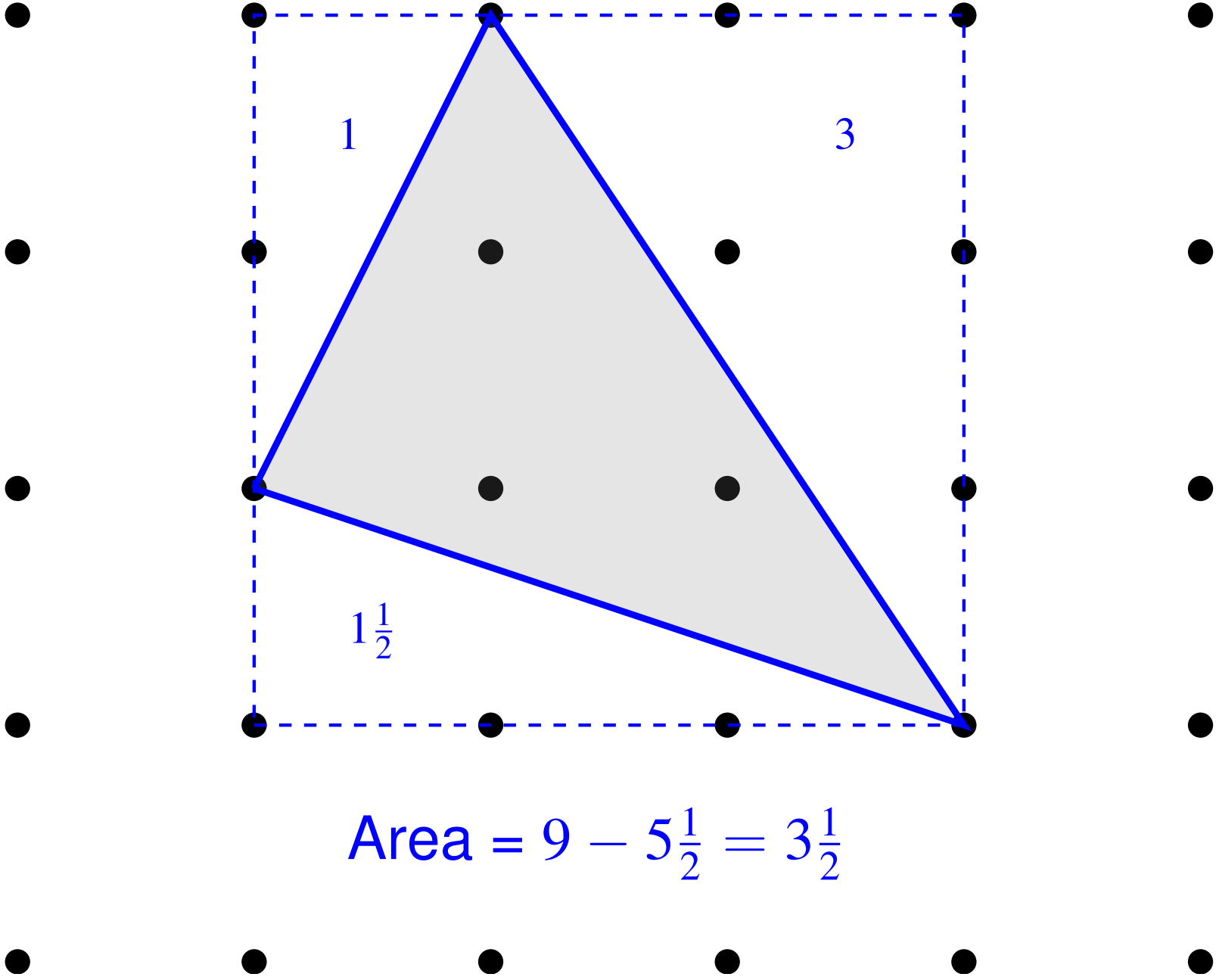


Area = 3 = $\frac{3 \times 2}{2}$









Area = $9 - 5\frac{1}{2} = 3\frac{1}{2}$

Geometrisches zur Zahlenlehre.[1]

Von

GEORG PICK.

Seit Gauss sind parallelogrammatische Gitter in der Ebene und entsprechende Raumfiguren vielfach zur Veranschaulichung und als heuristisches Mittel in der Zahlenlehre verwendet worden. Im Vergleich mit allen diesen Anwendungen verfolgen die nachfolgen Zeilen ein viel bescheideneres Ziel: es wird der Versuch gemacht, die Elemente der Zahlentheorie von vorn herein auf geometrische Basis zu stellen. Dazu dient eine trotz ihrer Einfachheit bisher, wie es scheint, unbemerkt gebliebene Flächenformel für Polygone, welche in ein Gitter eingezeichnet sind.

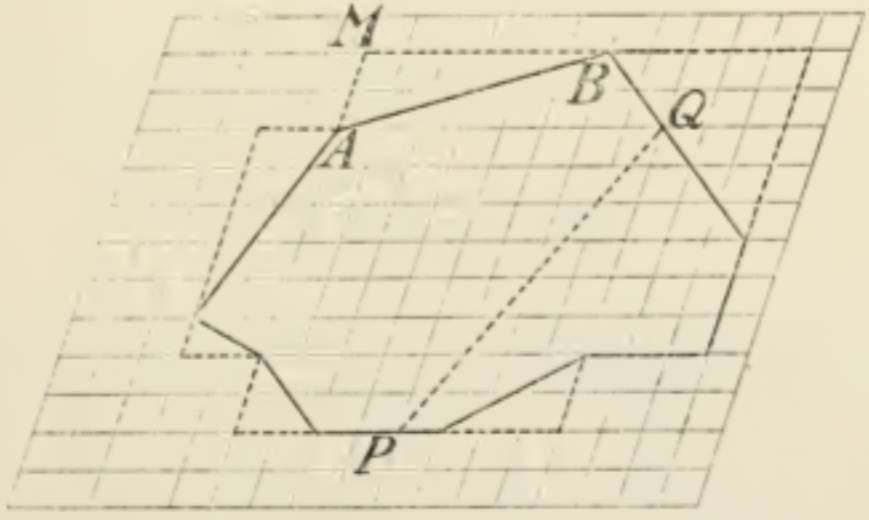
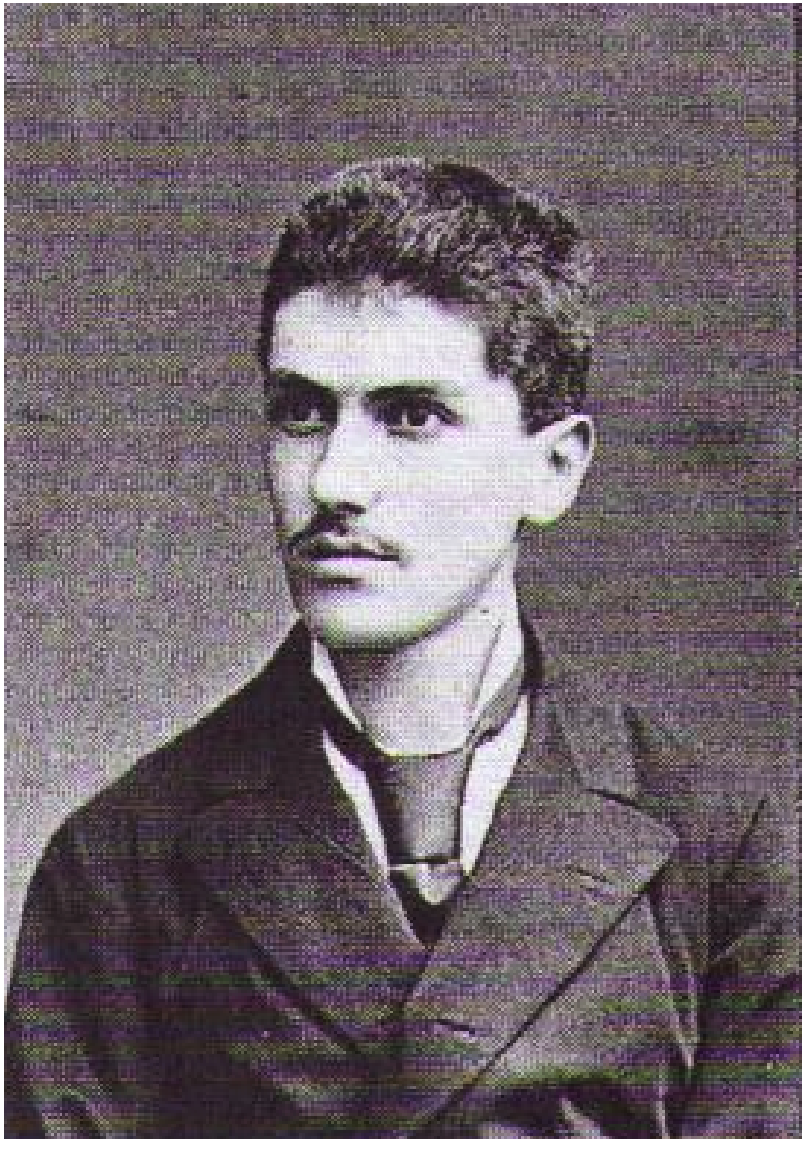


Fig. 2.



Georg Alexander Pick 1859–1942

A **lattice polygon** has all of its vertices on lattice points.

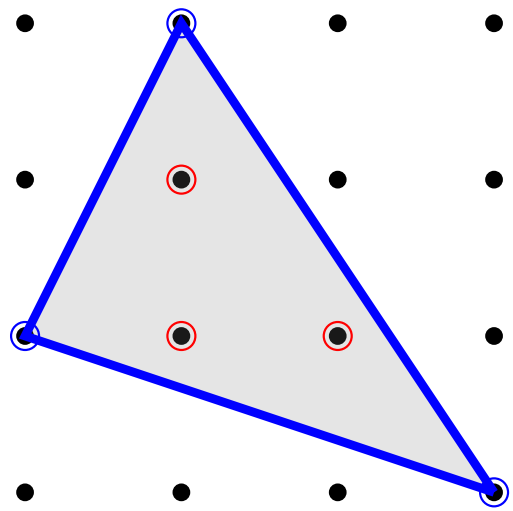
i = the number of lattice points strictly inside

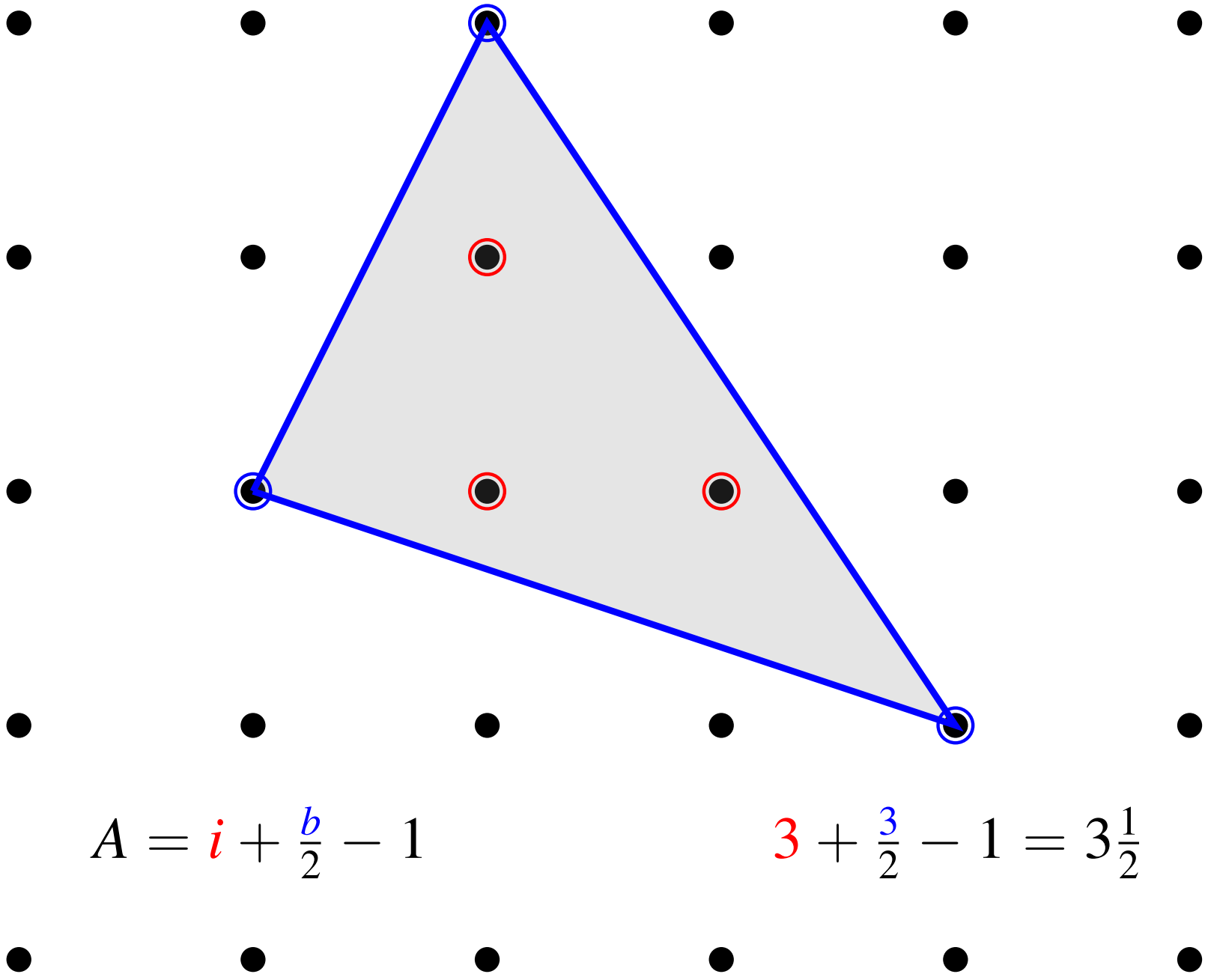
b = the number of lattice points on the perimeter

Pick's Theorem

The area of a lattice polygon is

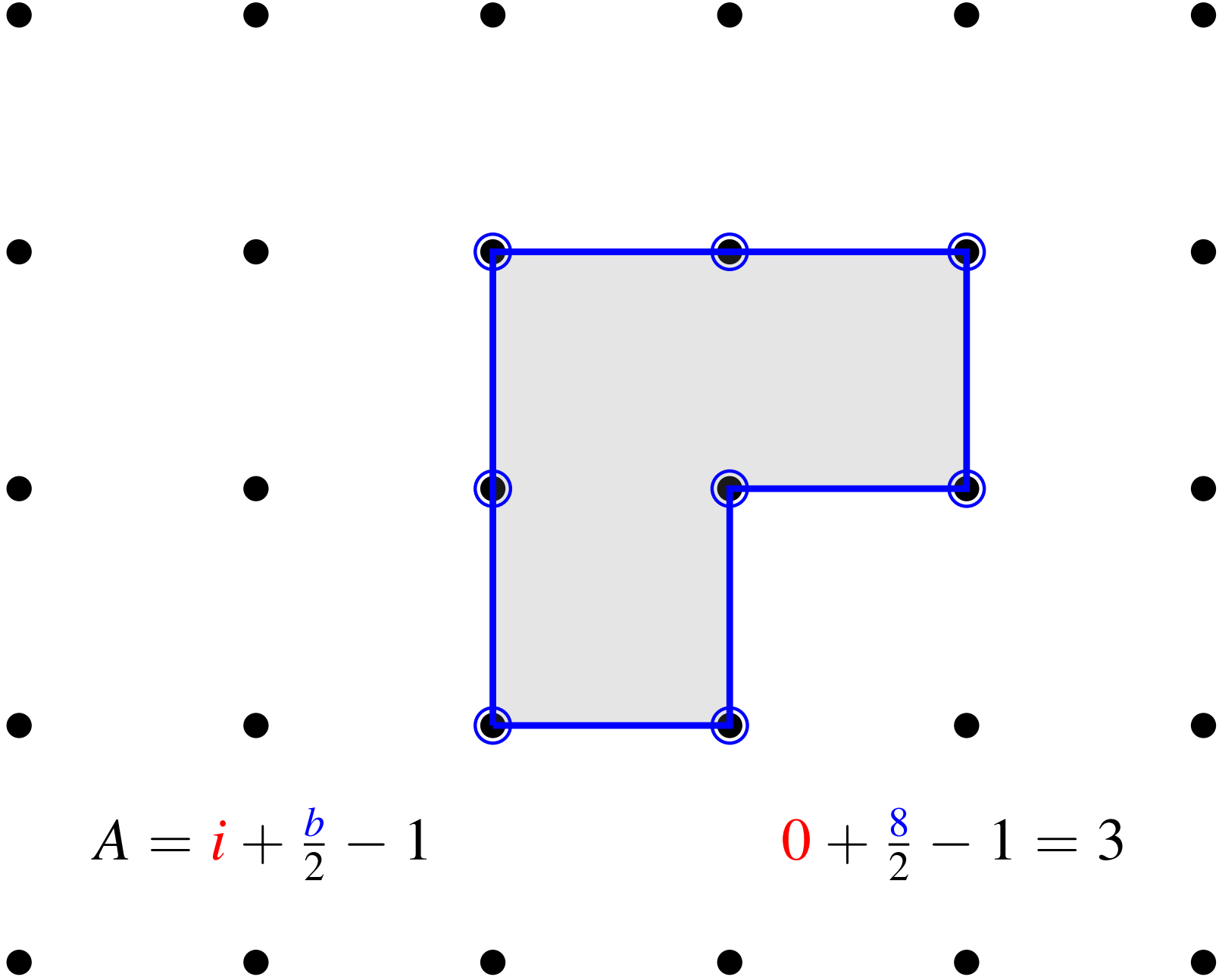
$$A = i + \frac{b}{2} - 1$$





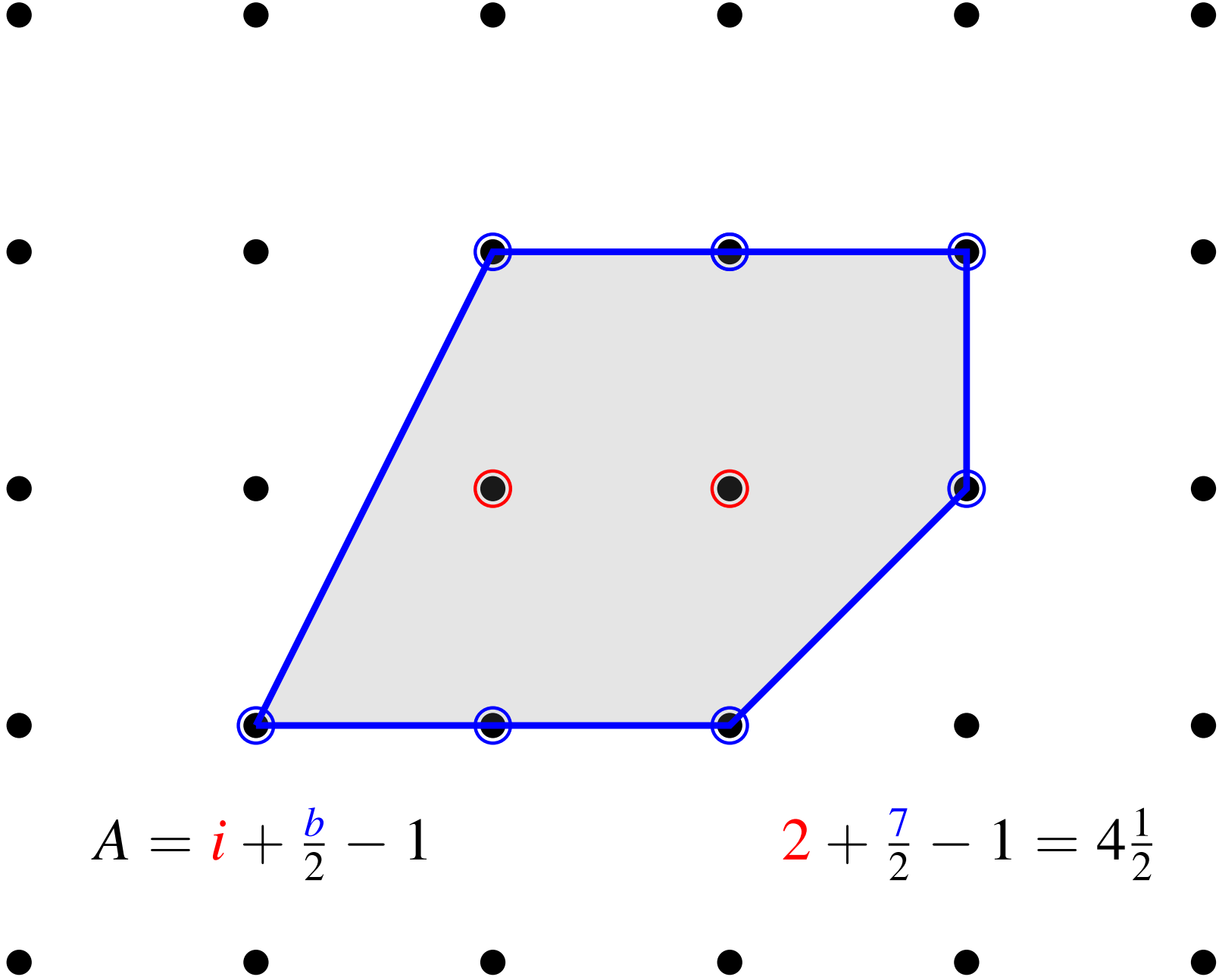
$$A = i + \frac{b}{2} - 1$$

$$3 + \frac{3}{2} - 1 = 3\frac{1}{2}$$



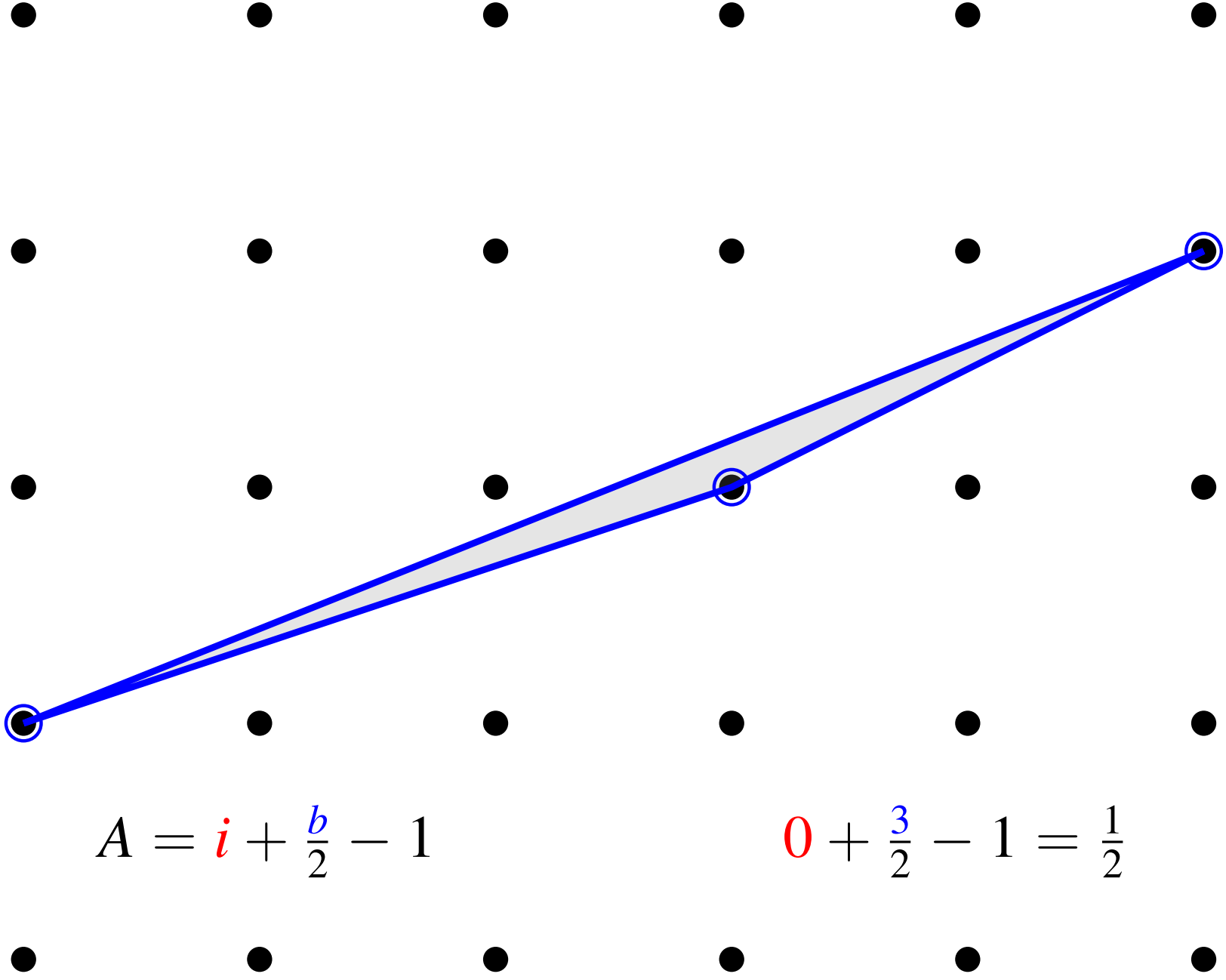
$A = i + \frac{b}{2} - 1$

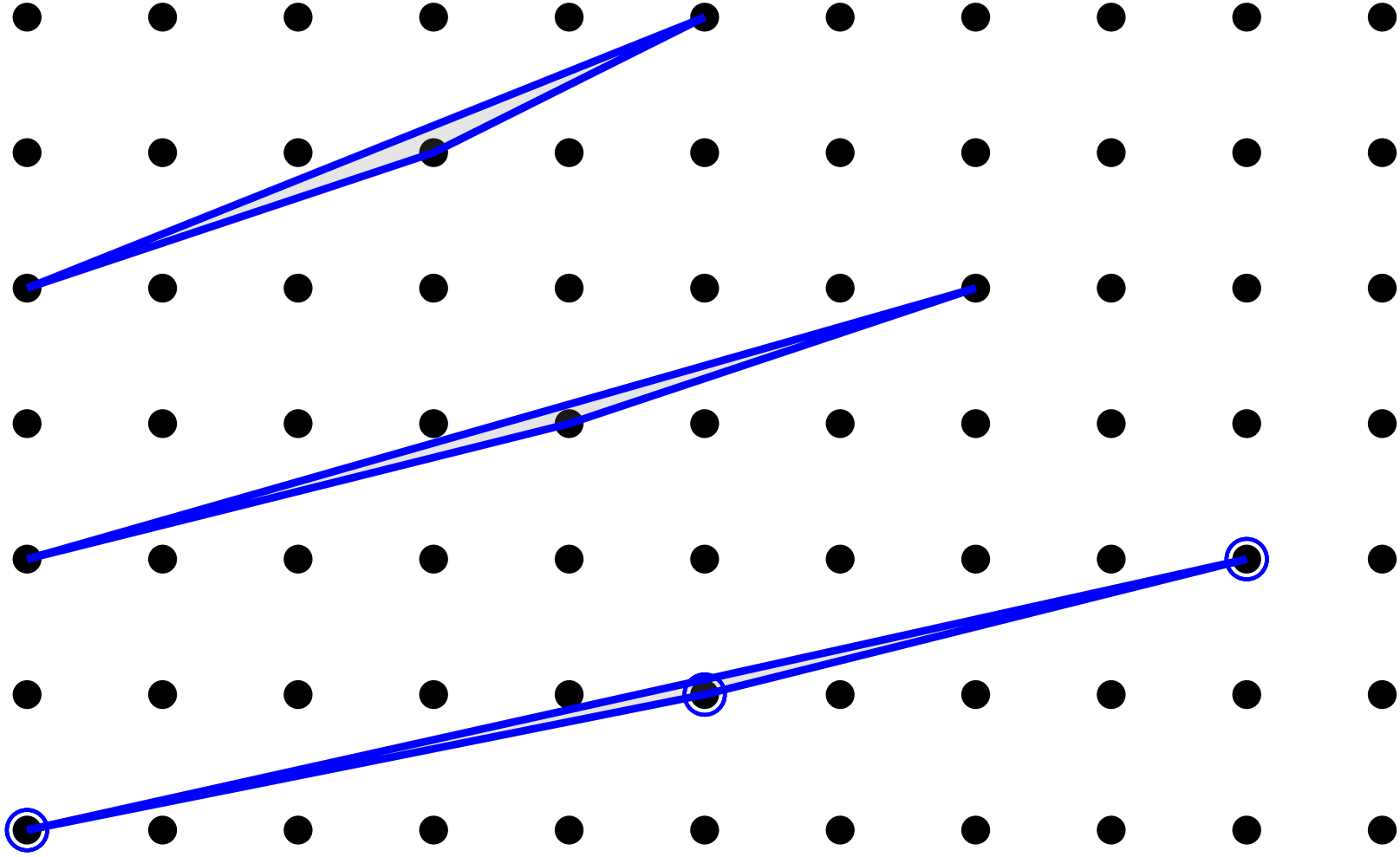
$0 + \frac{8}{2} - 1 = 3$



$A = i + \frac{b}{2} - 1$

$2 + \frac{7}{2} - 1 = 4\frac{1}{2}$

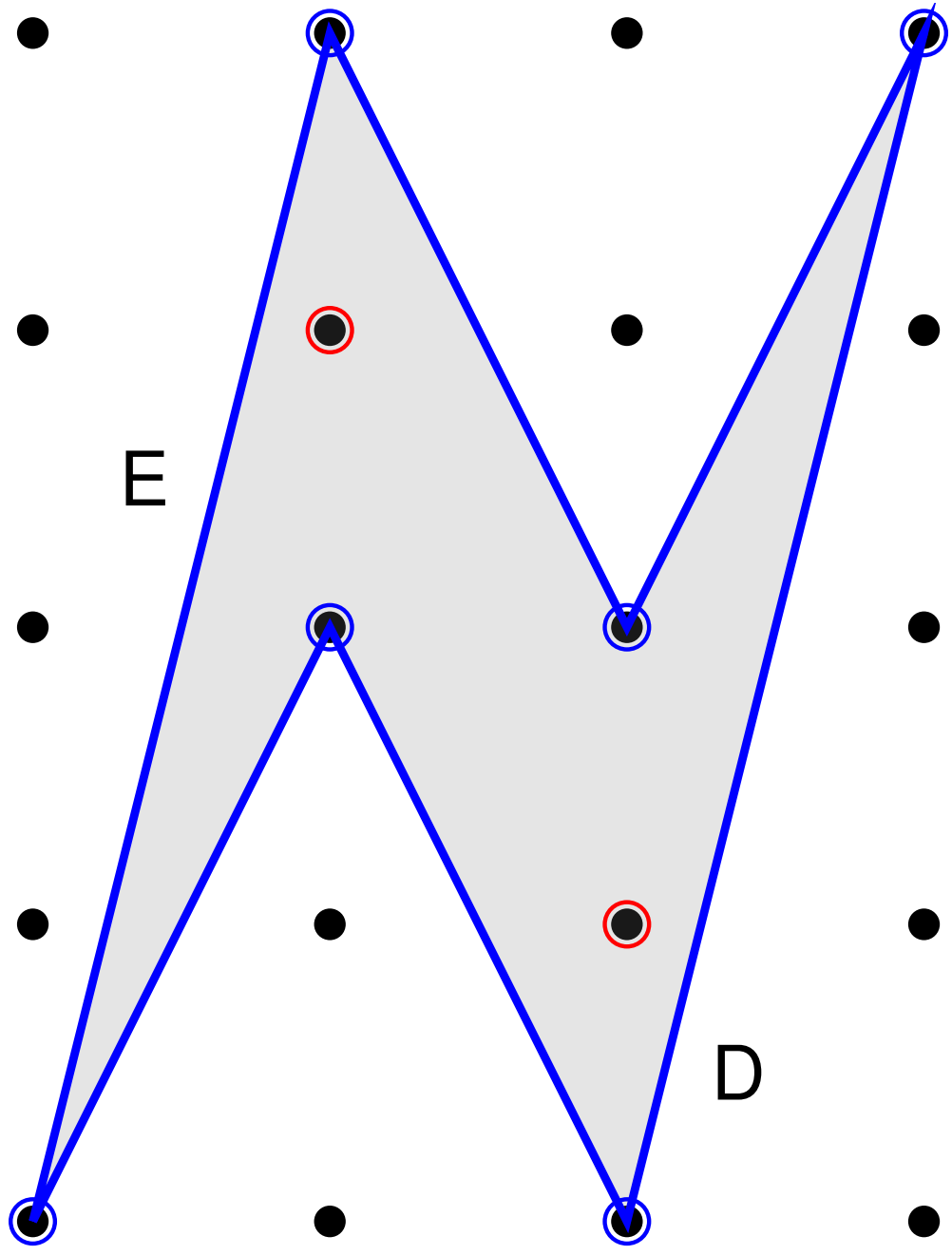

$A = i + \frac{b}{2} - 1$
$0 + \frac{3}{2} - 1 = \frac{1}{2}$



$A = i + \frac{b}{2} - 1$

$0 + \frac{3}{2} - 1 = \frac{1}{2}$

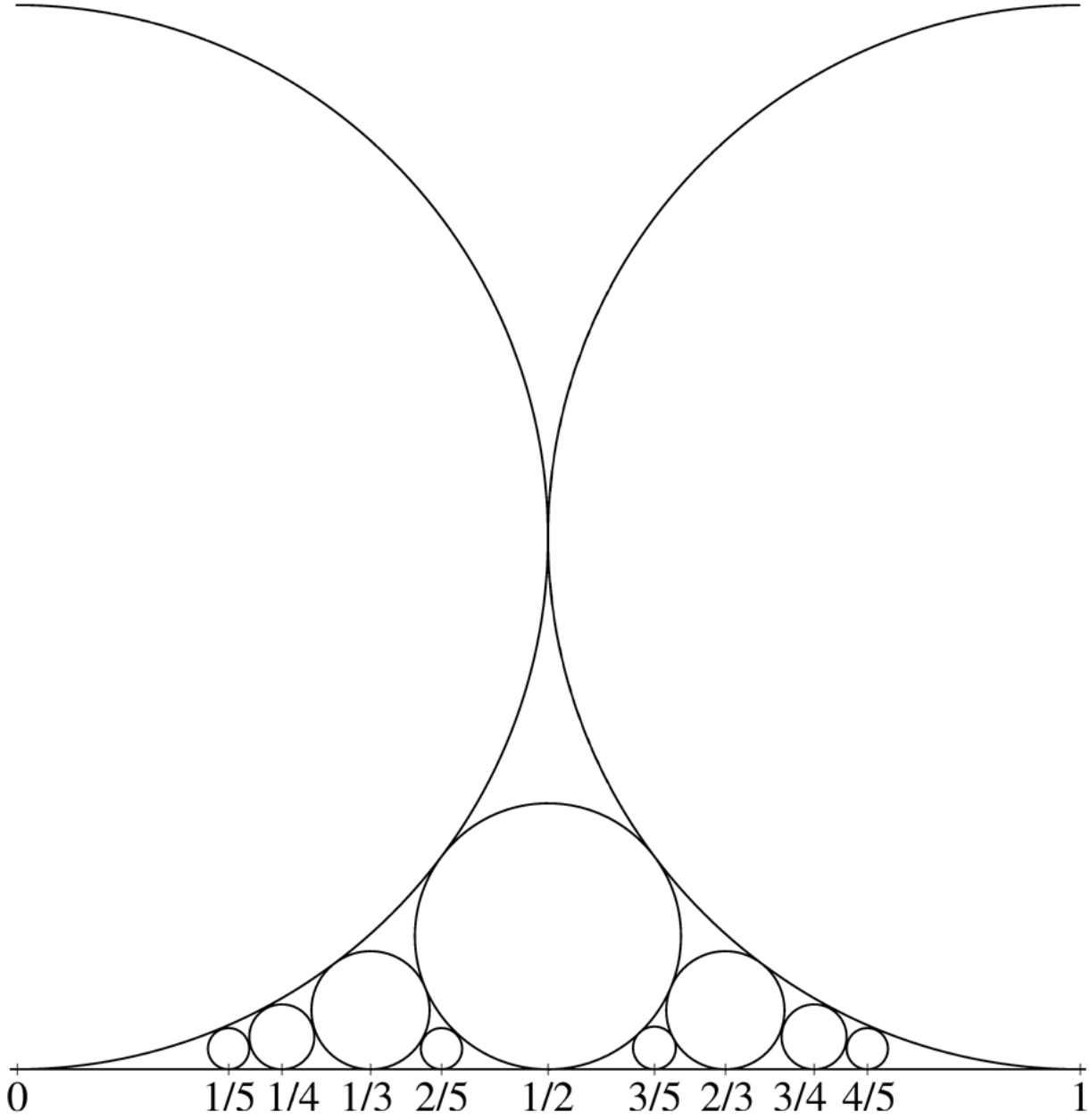
THE



$$A = i + \frac{b}{2} - 1$$

$$= 2 + \frac{6}{2} - 1$$

Ford circles



Farey sequences

$$\frac{0}{1}, \frac{1}{1}$$

$$\frac{0}{1}, \frac{1}{2}, \frac{1}{1}$$

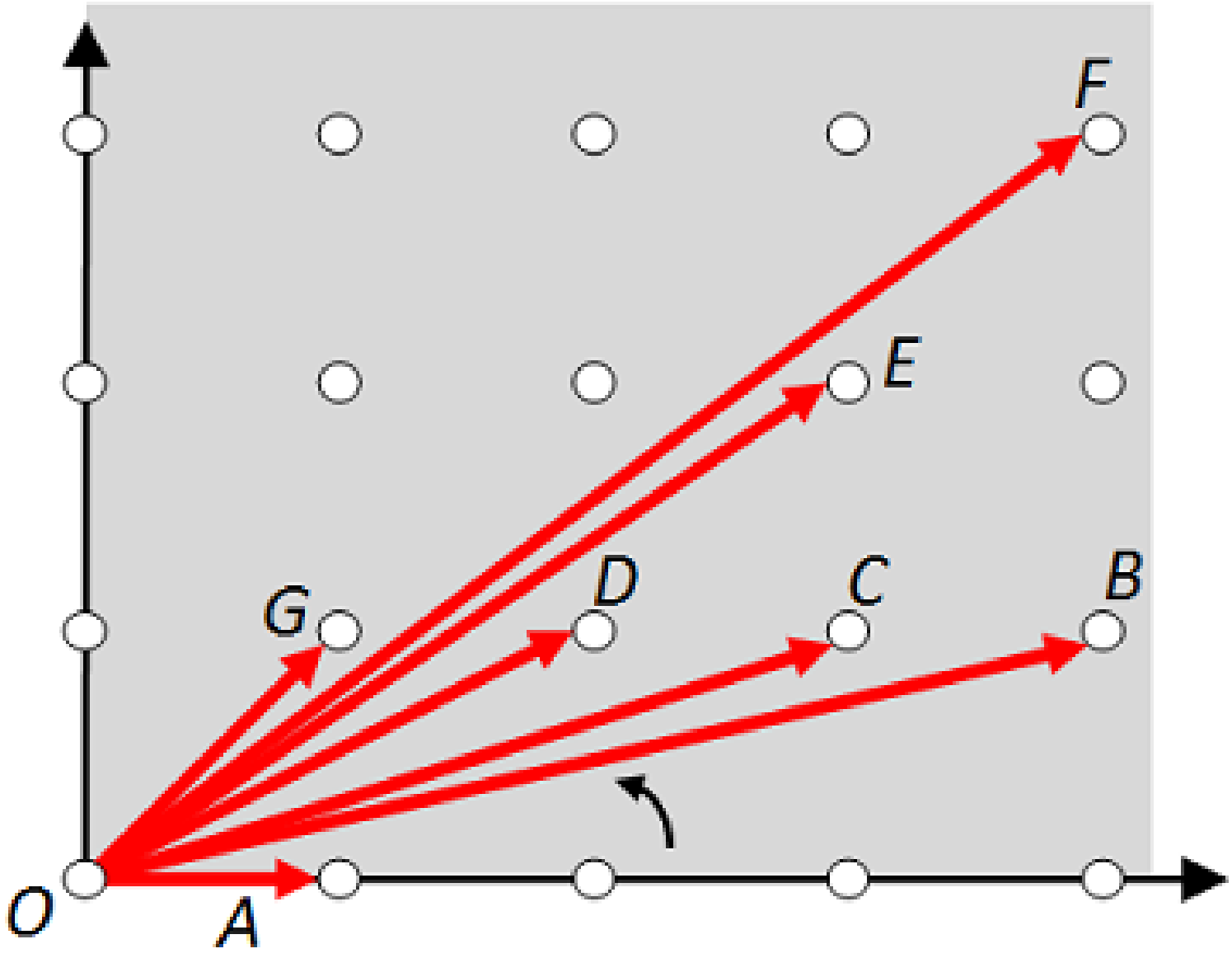
$$\frac{0}{1}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{1}{1}$$

$$\frac{0}{1}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{1}{1}$$

$$\frac{0}{1}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{1}{1}$$

$$\frac{0}{1}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{1}{1}$$

$$\frac{0}{1}, \frac{1}{7}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{2}{7}, \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{1}{2}, \frac{4}{7}, \frac{3}{5}, \frac{2}{3}, \frac{5}{7}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{1}{1}$$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