

Platonic Solids

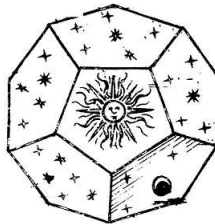
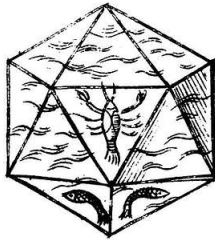
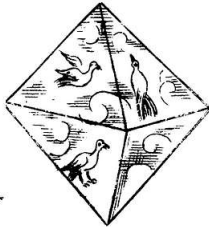
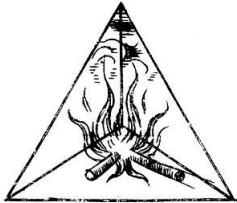
Asher Auel

Department of Mathematics
Yale University

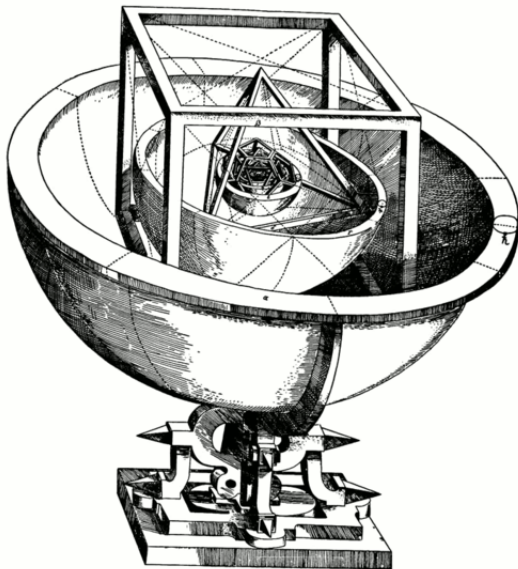
Resonance
December 2, 2017

Platonic solids

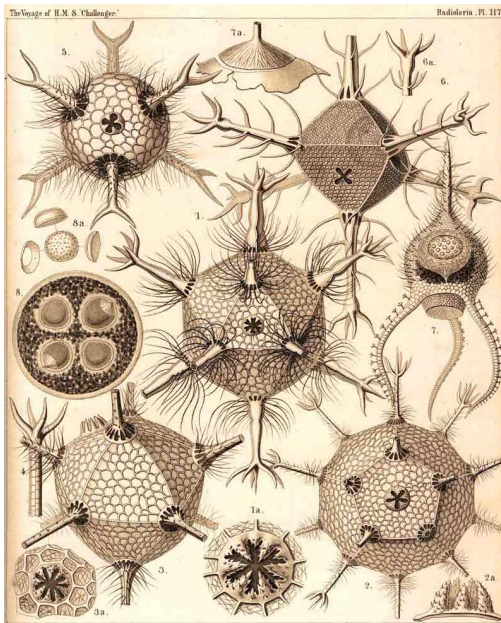
Elements



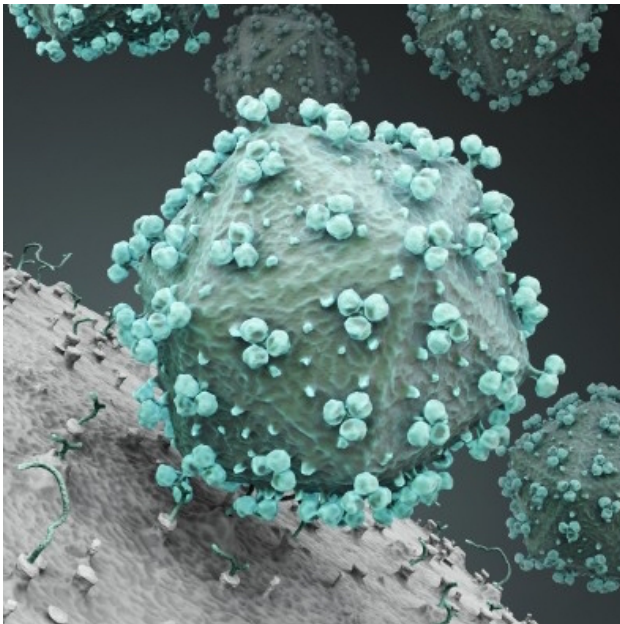
Kepler



Radiolaria



Virus



Platonic solids

3 triangles

4 triangles

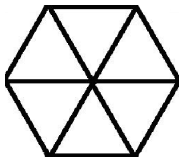
5 triangles

Platonic solids

3 triangles

4 triangles

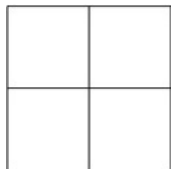
5 triangles



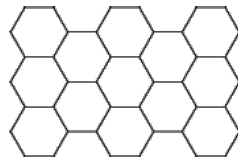
6 triangles

Platonic solids

3 squares

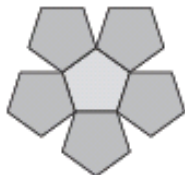


4 squares



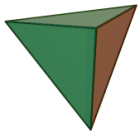
3 hexagons

3 pentagons

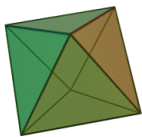


4 pentagons?

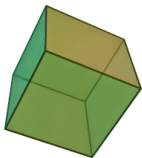
Platonic solids



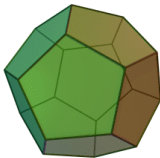
4 vertices
6 edges
4 faces



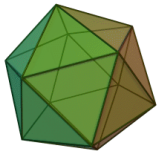
6 vertices
12 edges
8 faces



8 vertices
12 edges
6 faces

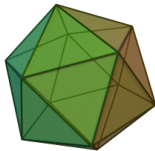
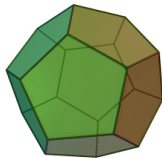
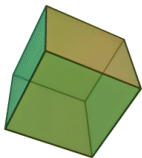
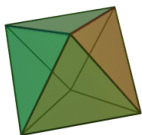
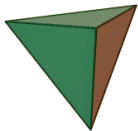


20 vertices
30 edges
12 faces



12 vertices
30 edges
20 faces

Platonic solids

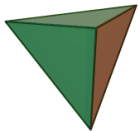


| | | | | |
|------------|------------|------------|-------------|-------------|
| 4 vertices | 6 vertices | 8 vertices | 20 vertices | 12 vertices |
| - 6 edges | - 12 edges | - 12 edges | - 30 edges | - 30 edges |
| + 4 faces | + 8 faces | + 6 faces | + 12 faces | + 20 faces |
| <hr/> | <hr/> | <hr/> | <hr/> | <hr/> |
| = 2 | = 2 | = 2 | = 2 | = 2 |

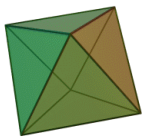
$$V - E + F = 2$$

Euler characteristic

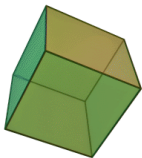
Platonic solids



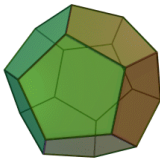
4 vertices
6 edges
4 faces



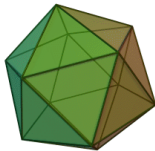
6 vertices
12 edges
8 faces



8 vertices
12 edges
6 faces



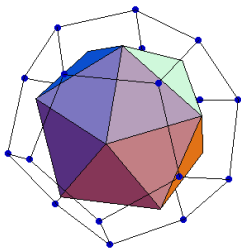
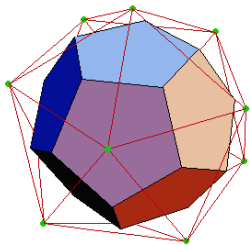
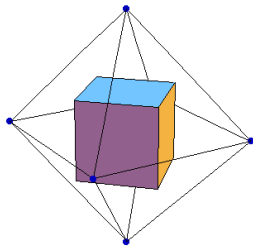
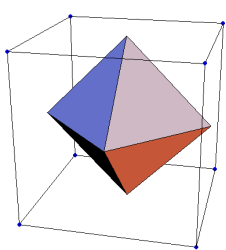
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30 edges
12 faces



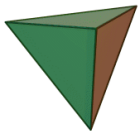
12 vertices
30 edges
20 faces

Duality

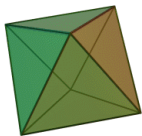
Platonic solids



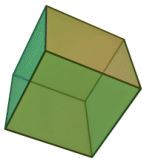
Platonic solids



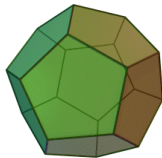
4 vertices
6 edges
4 faces



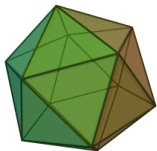
6 vertices
12 edges
8 faces



8 vertices
12 edges
6 faces



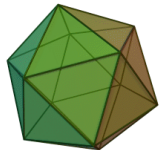
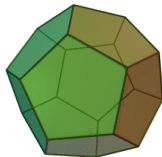
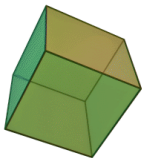
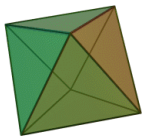
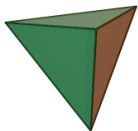
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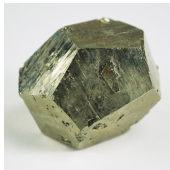
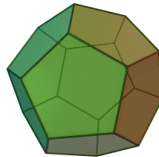
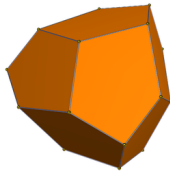
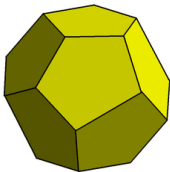
12 vertices
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20 faces

Platonic solids

Platonic solids in crystals



Nonregular dodecahedral crystals

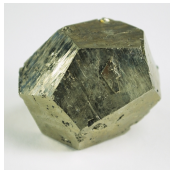
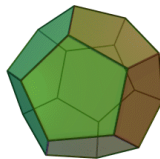
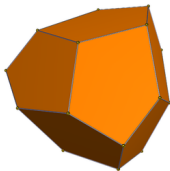
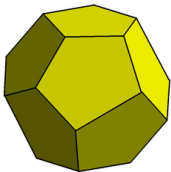


Pyritohedron



Tetartoid

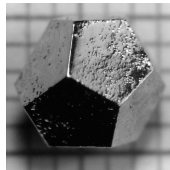
Nonregular dodecahedral crystals



Pyritohedron

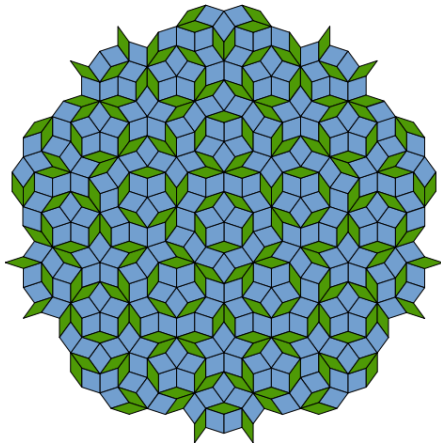
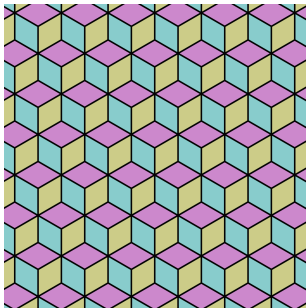


Tetartoid

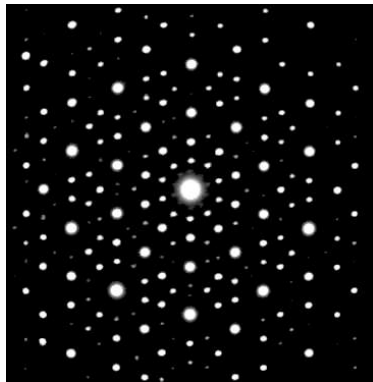
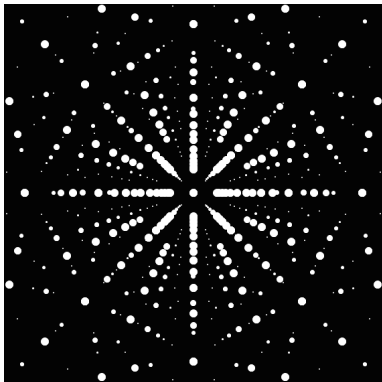


Quasicrystal

Crystals and quasicrystals



Crystals and quasicrystals



Archimedean solids