Platonic Solids and Symmetry

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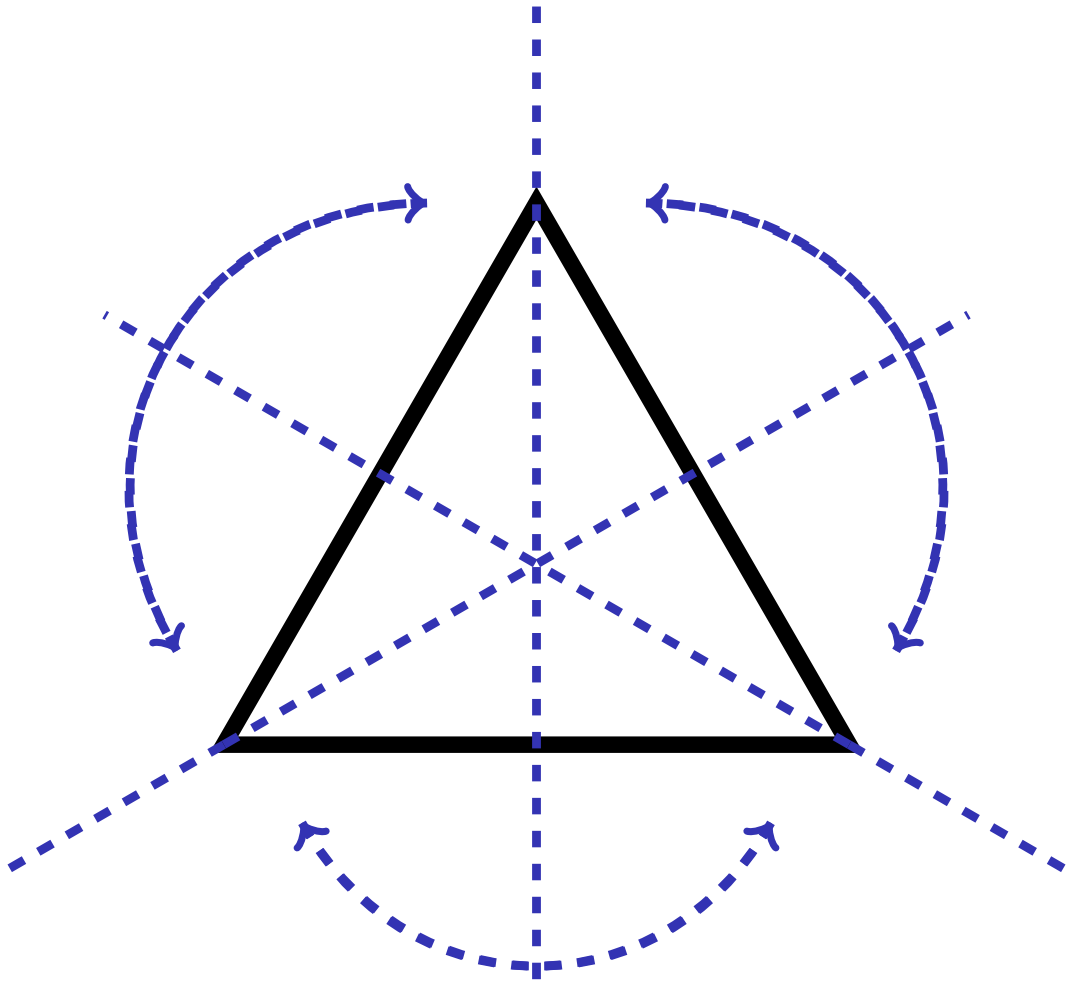
Department of Mathematics
Yale University

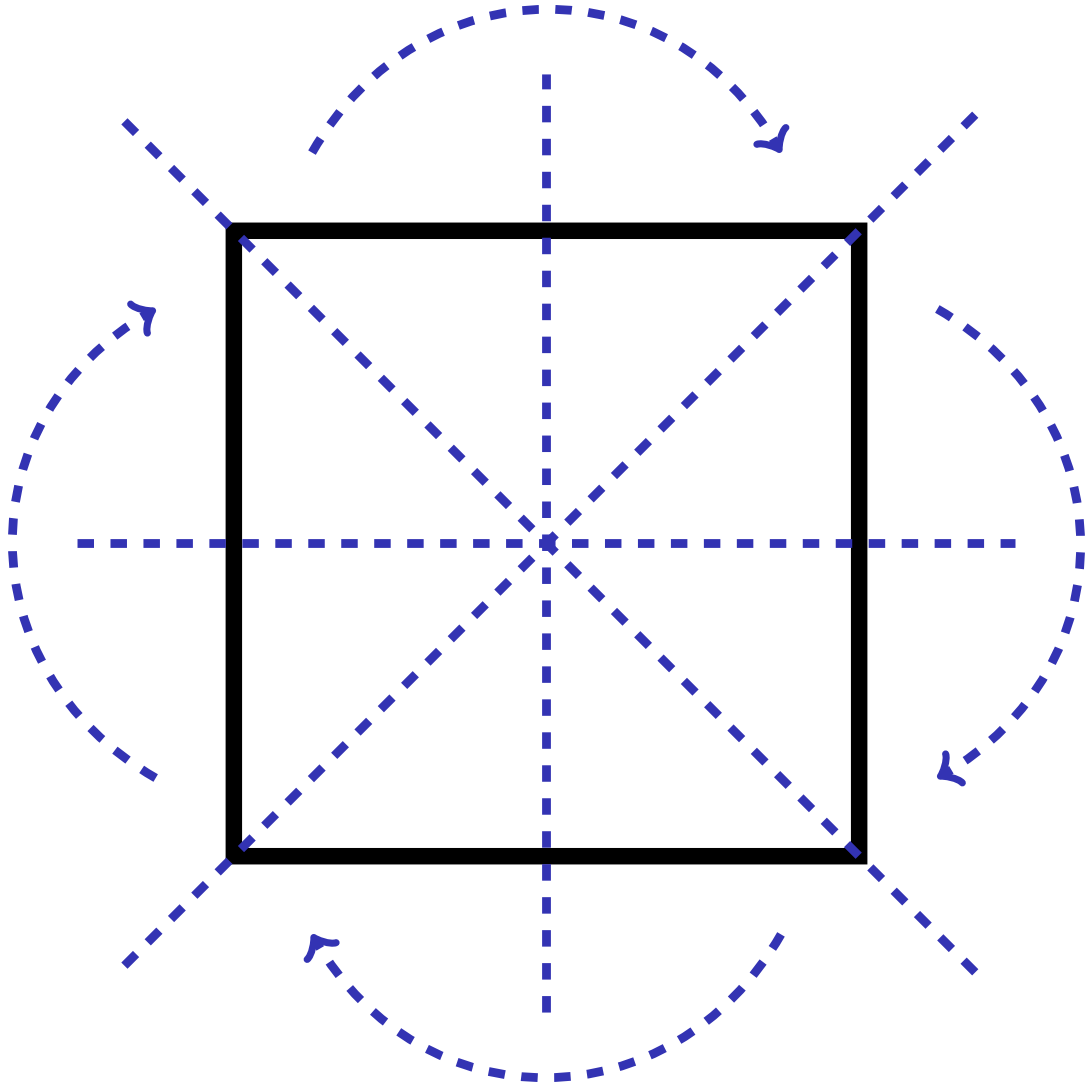
Math Mornings
October 16th, 2016

Triangle



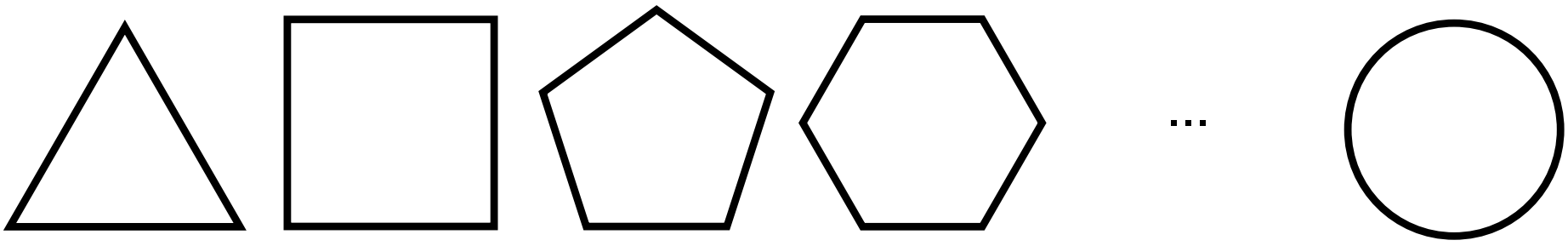
How many symmetries?

Square



How many symmetries?

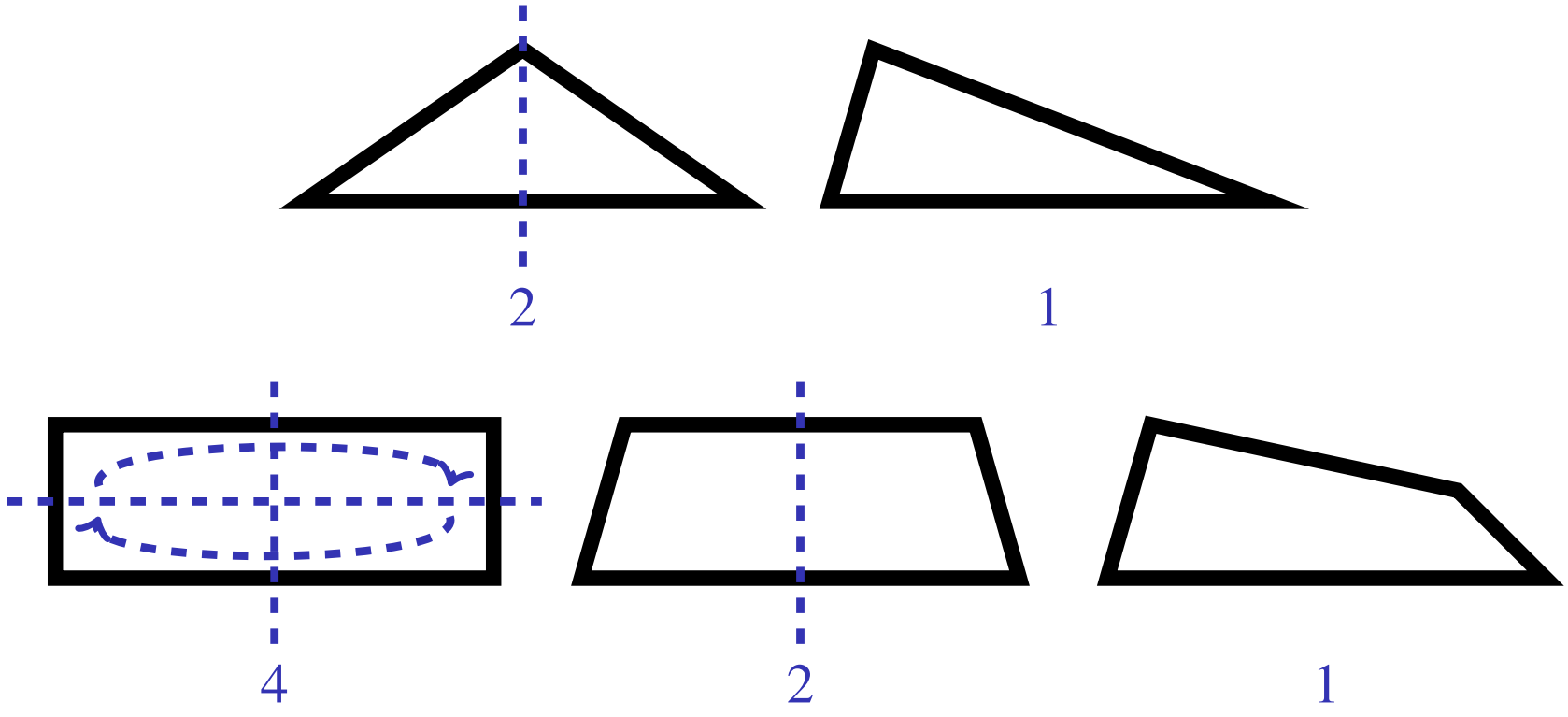
Regular n-gons



$n = 3$ $n = 4$ $n = 5$ $n = 6$ ... $n = \infty$

$2 \times 6 = 6$ $2 \times 8 = 4$ $2 \times 10 = 5$ $2 \times 12 = 6$ $2 \times \infty$

Geometry from Symmetry



Regular n-gons are distinguished by their number of symmetries

Principle. Symmetry determines geometry!

Erlangen Program

Felix Klein 1849–1925

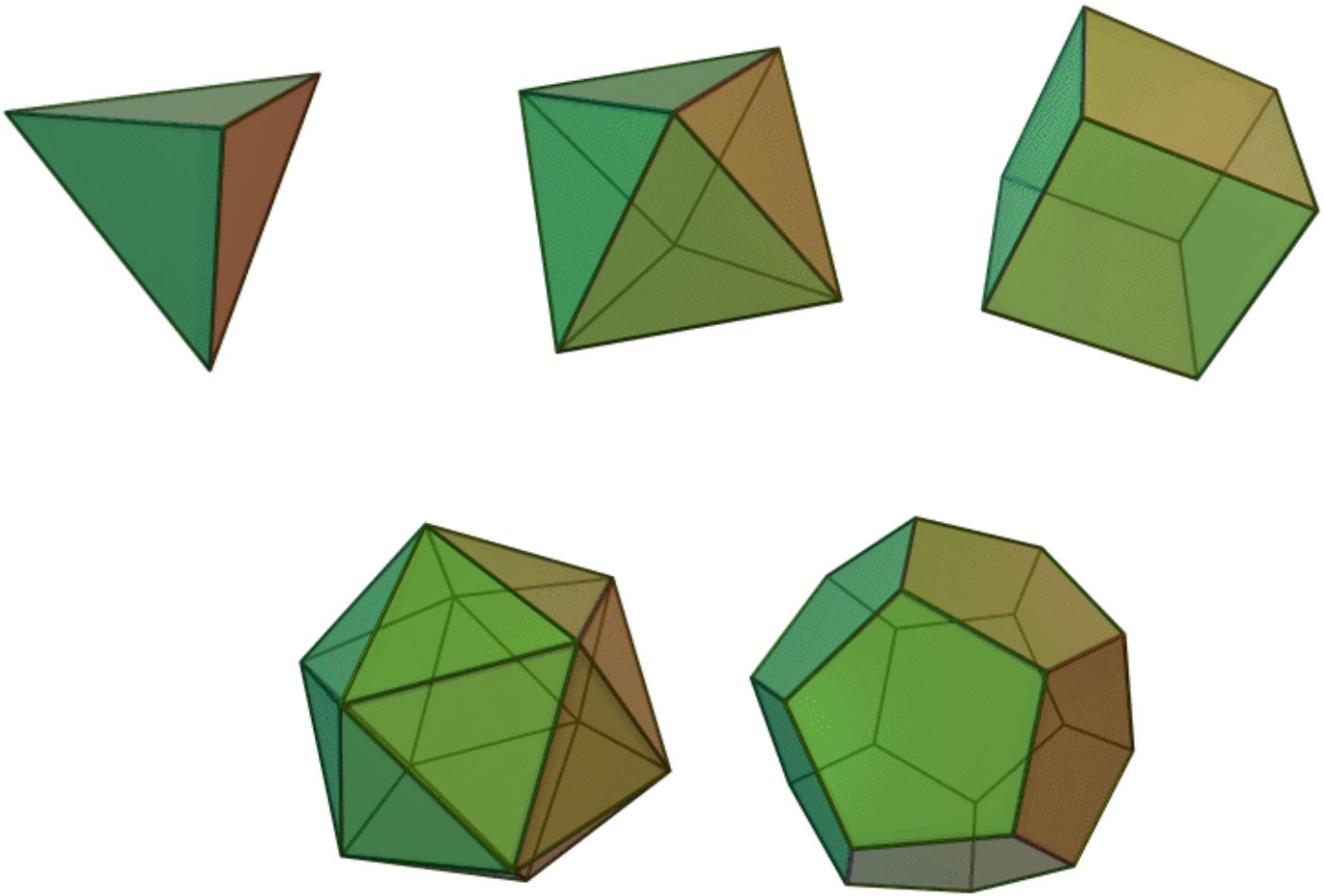
Dr. Felix Klein,
o. ö. Professor der Mathematik an der Universität Erlangen.

Programm

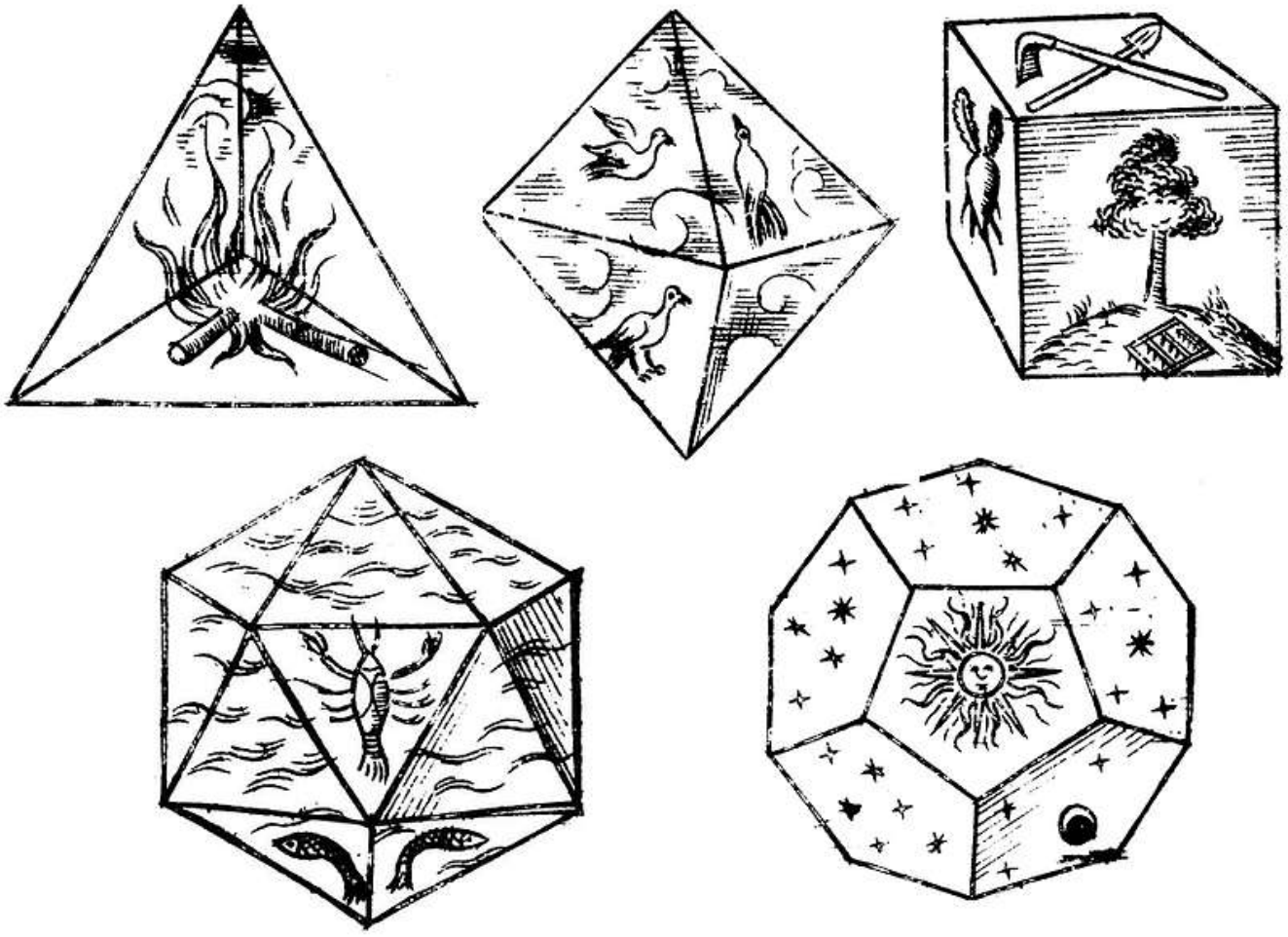
zum Eintritt in die philosophische Facultät und den Senat
der k. Friedrich-Alexanders-Universität
zu Erlangen.

Inaugural lecture 1872

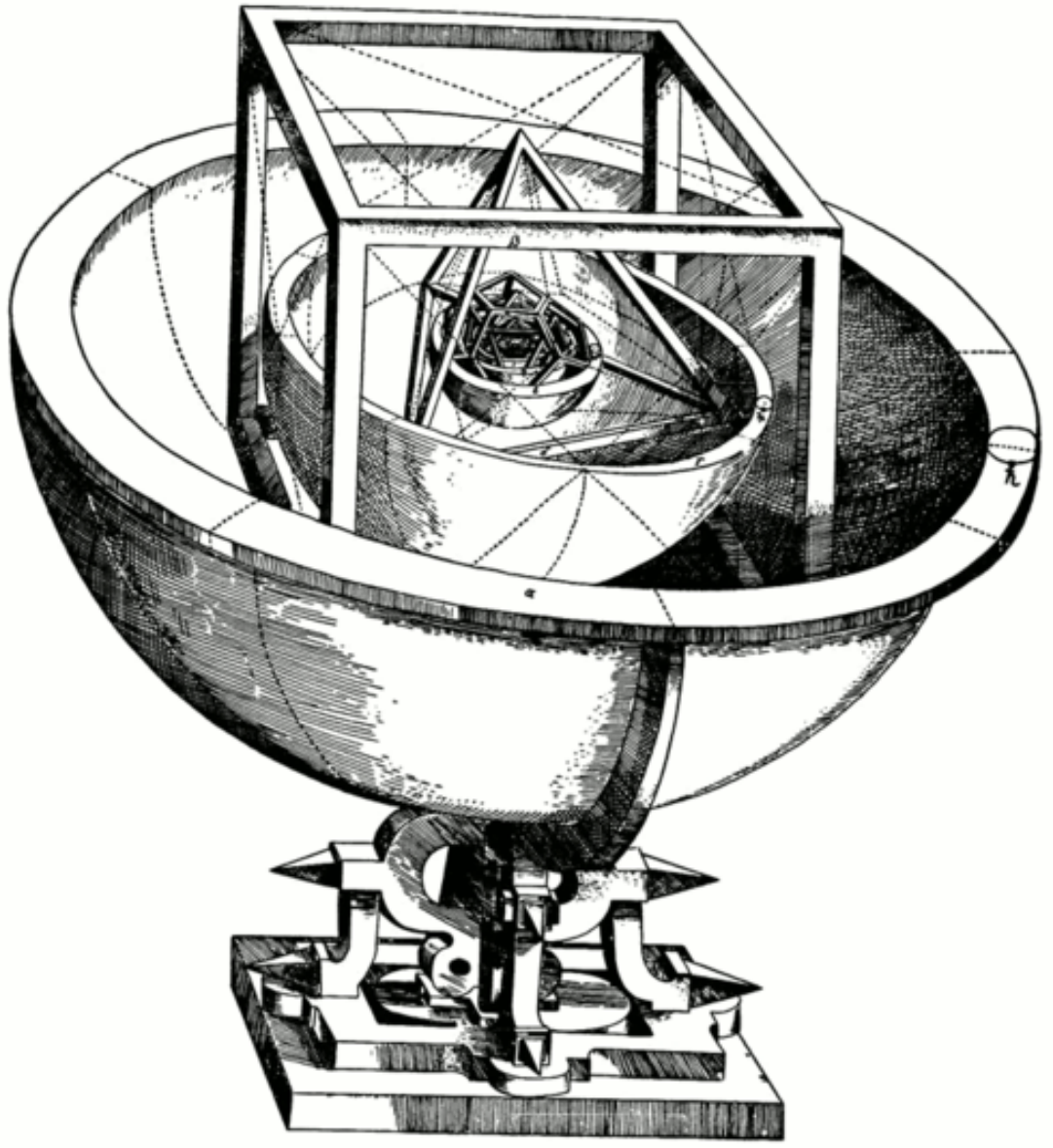
Platonic Solids



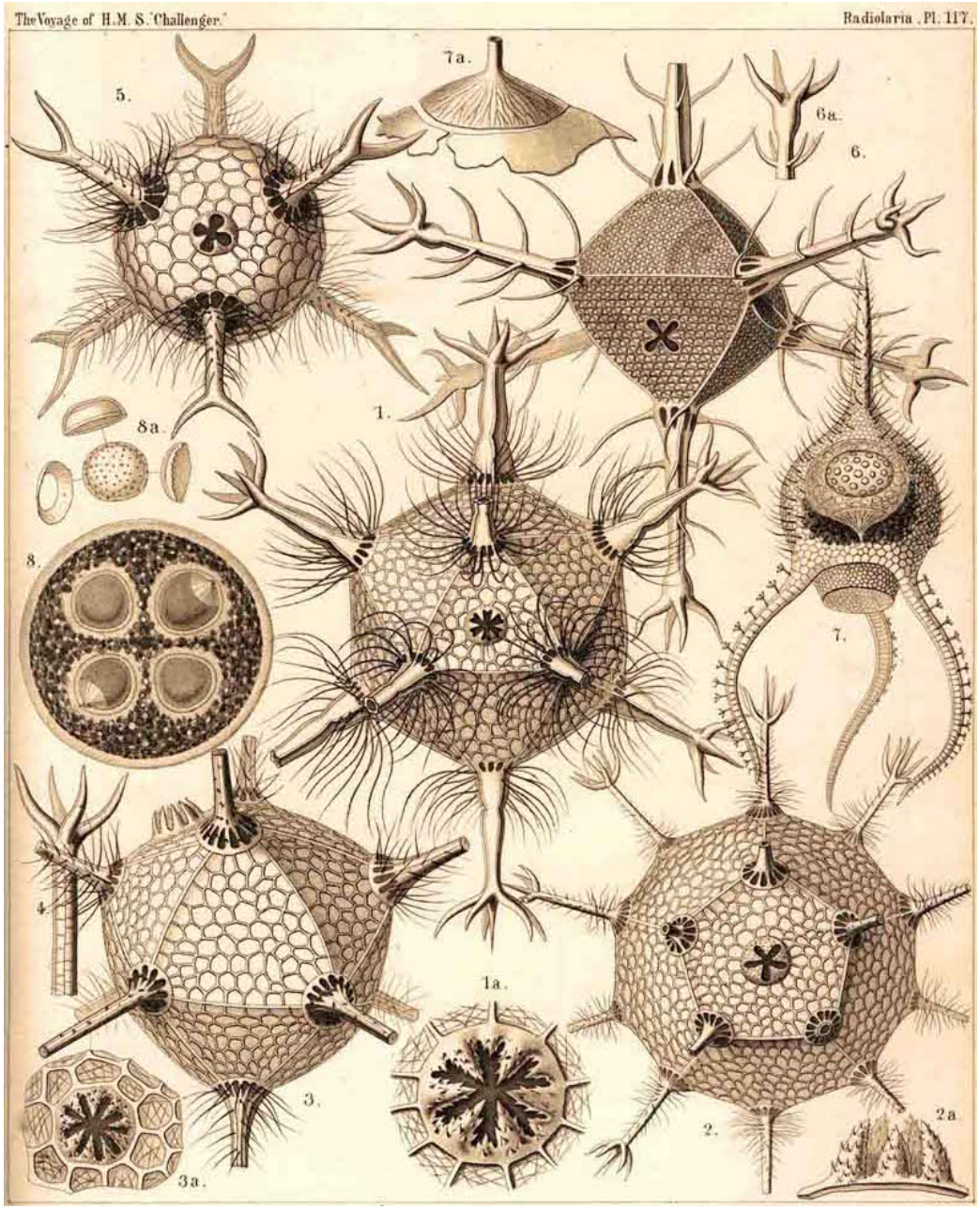
Platonic Solids



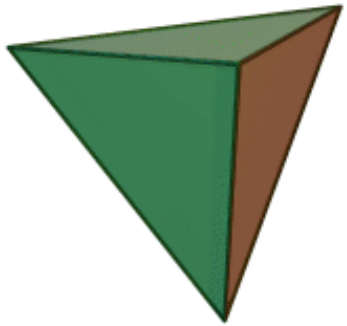
Platonic Solids



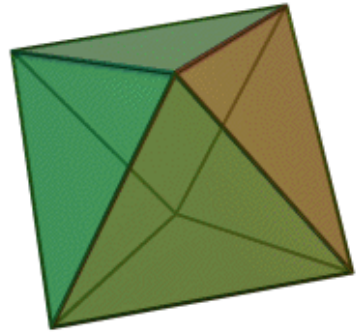
Platonic Solids



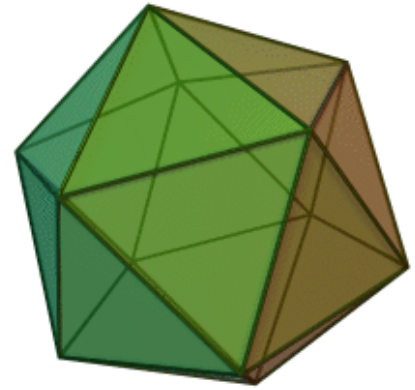
Platonic Solids



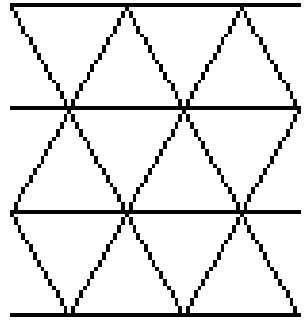
3 triangles



4 triangles

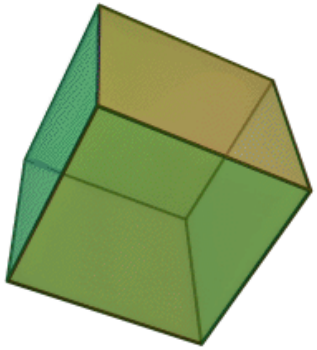


5 triangles

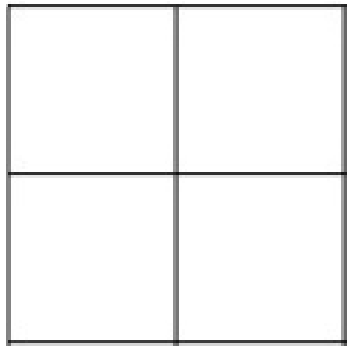


6 triangles

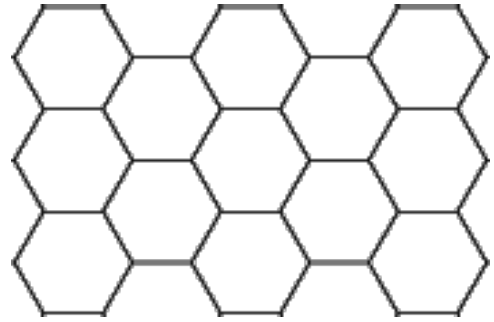
Platonic Solids



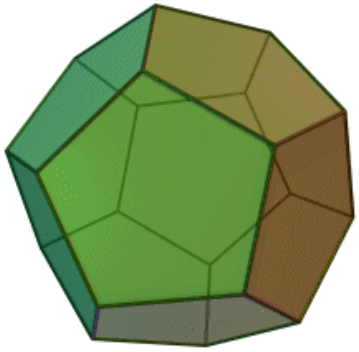
3 squares



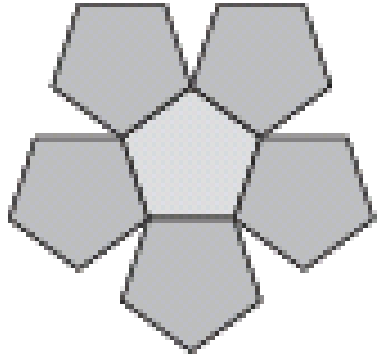
4 squares



6 hexagons

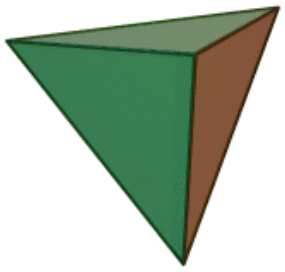
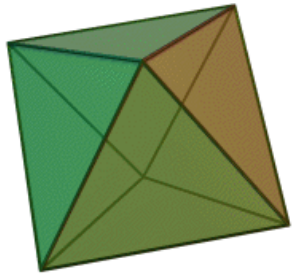
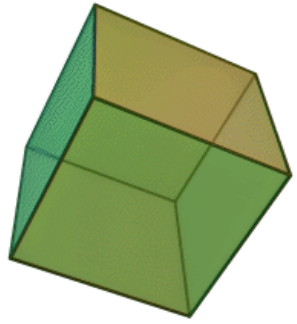
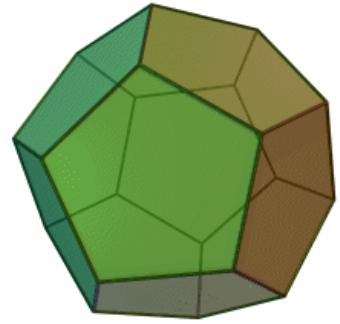
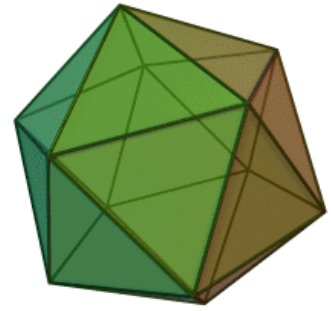


5 pentagons



6 pentagons?

Platonic Solids

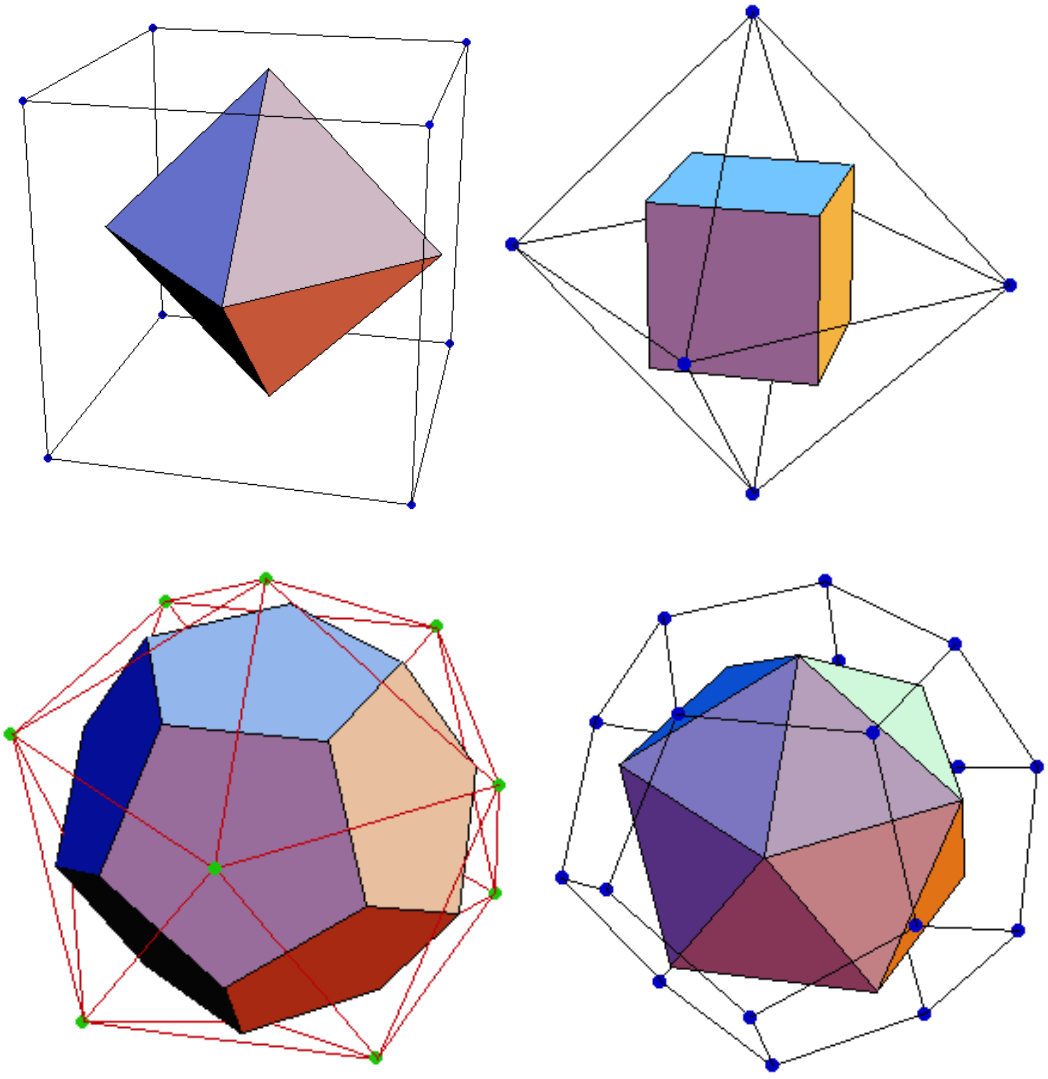
4 vertices	6 vertices	8 vertices	20 vertices	12 vertices
$-$ 6 edges	$-$ 12 edges	$-$ 12 edges	$-$ 30 edges	$-$ 30 edges
$+$ 4 faces	$+$ 8 faces	$+$ 6 faces	$+$ 12 faces	$+$ 20 faces
$=2$	$=2$	$=2$	$=2$	$=2$

$$V - E + F = 2$$

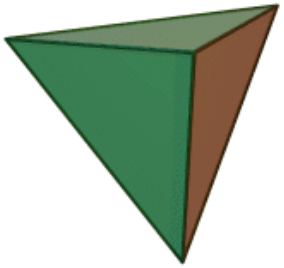
Euler characteristic

Duality

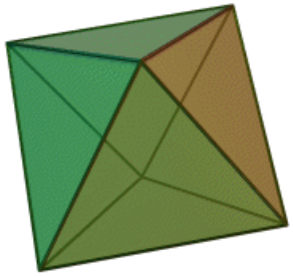
Platonic Solids



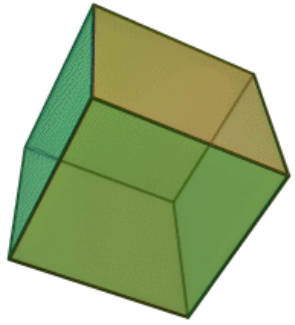
Platonic Solids



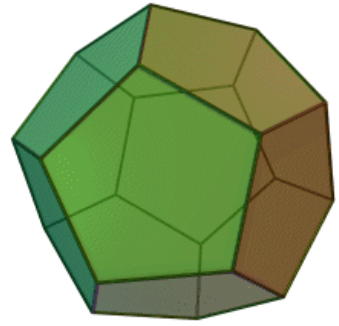
4 vertices
6 edges
4 faces



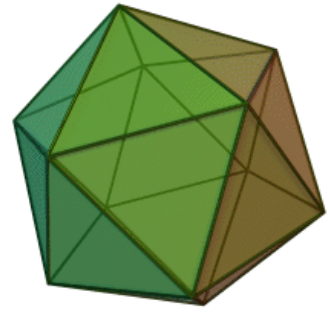
6 vertices
12 edges
8 faces



8 vertices
12 edges
6 faces

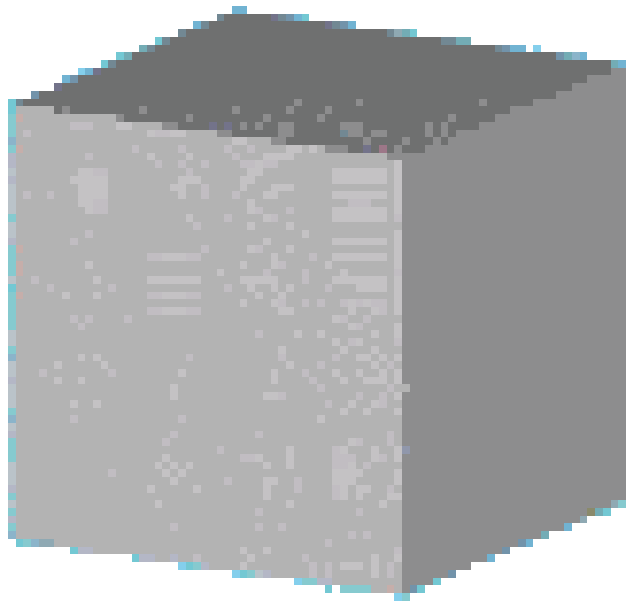


20 vertices
30 edges
12 faces



12 vertices
30 edges
20 faces

Platonic Solids



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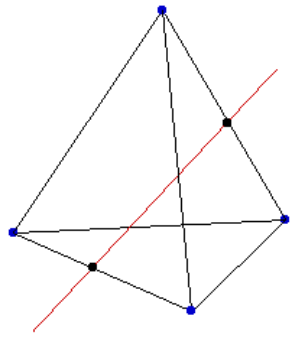

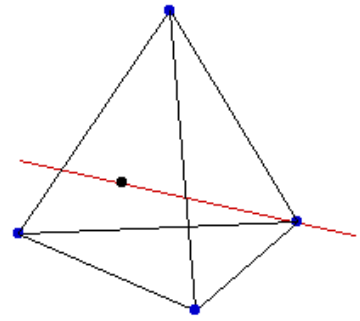

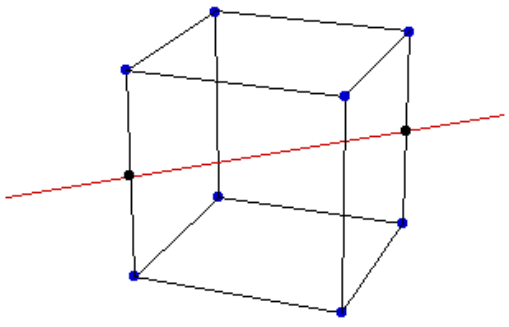

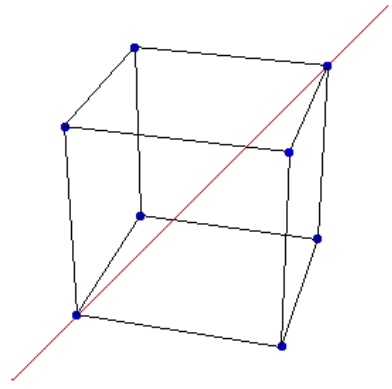

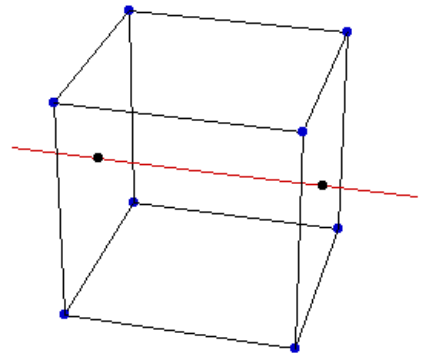

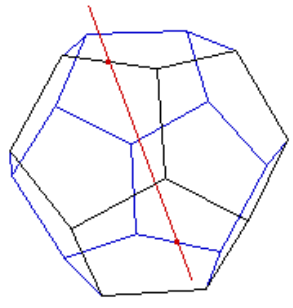

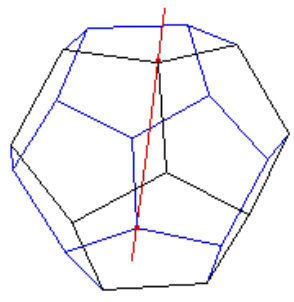

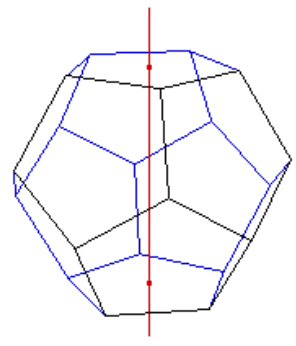

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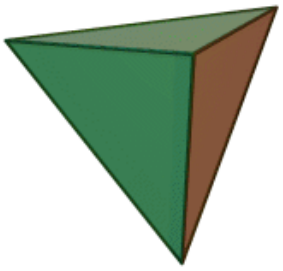
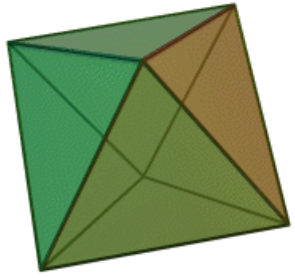


$$
\begin{array}{r}
1 \\
3 \times 1 \\
4 \times 2 \\
\hline
12
\end{array}
$$



$$
\begin{array}{r}
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6 \times 1 \\
4 \times 2 \\
3 \times 3 \\
\hline
24
\end{array}
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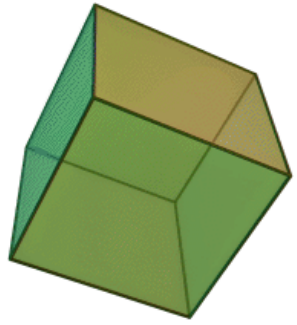

$$
\begin{array}{r}
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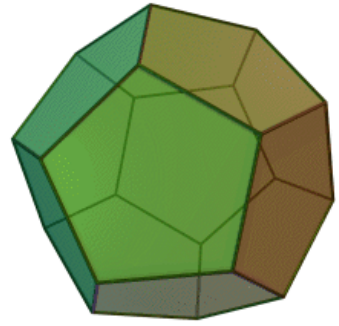
6 \times 1 \\
4 \times 2 \\
3 \times 3 \\
\hline
24
\end{array}
$$



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6 \times 4 \\
\hline
60
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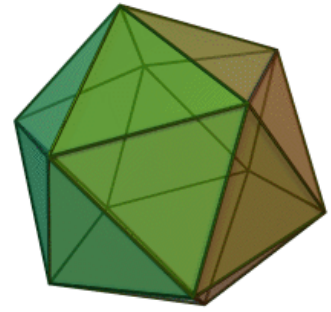

$$
\begin{array}{r}
1 \\
15 \times 1 \\
10 \times 2 \\
6 \times 4 \\
\hline
60
\end{array}
$$