# Brill-Noether theory via $K3$ Surfaces

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*Abstract*

Classical Brill–Noether (BN) theory concerns the geometry of genus $g$ curves admitting a linear system of degree $d$ and dimension $r$. When the BN number is negative, the locus of such curves is a proper subvariety of the moduli space of genus $g$ curves, called a special BN locus. Focusing on curves on $K3$ surfaces, we'll summarize a strategy for distinguishing special BN loci by studying the lifting of line bundles on the curve. This allows us to relate the problem of distinguishing BN loci to lattice theoretic conditions on the Picard group of $K3$ surfaces. We'll state our Maximal Brill-Noether loci conjecture and verify the conjecture in genus 9–19, 22, and 23. We prove new lifting results for rank 3 linear systems by studying the stability of Lazarsfeld–Mukai bundles that allow us to prove these novel results. This is joint work with Asher Auel.