# Finite-type invariants of order one for framed and long virtual knots

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*Abstract*

There are no Vassiliev invariants of degree one for classical knots. A. Henrich proved the existence of a sequence of three Vassiliev invariants of degree one for virtual knots. The invariants get stronger and stronger, and the final invariant is universal, in that we can reconstruct any other degree one Vassiliev invariant using a certain natural construction. The proof of these results extended Turaev's notion of a based matrix to the class of flat singular virtual knots with one double point. We generalize Henrich's results to the cases of framed virtual knots and long virtual knots, both framed and unframed. This requires us to extend the notion of based matrix to the framed case and to the long case. We also discuss the behavior of the invariants for long knots under connected sum and closure.