Algebras Counting Self-Intersections of Curves

Patricia Cahn

Abstract

In this thesis, we discuss an algebraic method of computing the minimum number of self-intersection points of a curve in a given free homotopy class. This work was motivated by a conjecture of Turaev. Namely, Turaev defined a Lie cobracket on the vector space generated by the nontrivial free homotopy classes of loops on a surface, and conjectured that the cobracket of a class is zero if and only if the class contains a power of a simple loop. While Le Donne showed that Turaev's conjecture holds on surfaces of genus zero, Chas found examples which demonstrate that Turaev's conjecture is not true in general. One can construct a generalization of Turaev's cobracket in the spirit of the Andersen-Mattes-Reshetikhin Algebra of chord diagrams. We show Turaev's conjecture is true when formulated for this generalized cobracket. We also show that the generalized cobracket yields an algebraic expression for the minimal self-intersection number. Turaev defined his cobracket for virtual strings as well as for curves on surfaces, and the generalized cobracket can also be defined for virtual strings. We discuss cases when this generalized cobracket yeilds an expression for the minimal self-intersection number of a virtual string. We also compare a bound on the minimal self-intersection number of a virtual string given by the generalized cobracket to a bound given by Turaev's based matrix invariant, and construct an example where the generalized cobracket gives a better estimate than the based matrix does.