Refocusing of Null-Geodesics in Lorentz Manifolds

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Abstract: We investigate weak and strong refocusing of light rays in a space-time and related concepts. A strongly causal space-time (X^{n+1}, g) is *strongly refocusing at* $x \in X$ if there is a point $y \neq x$ such that all null-geodesics through y pass through x. A space-time is *strongly refocusing* if it is strongly refocusing at some point.

Robert Low introduced three definitions of *(weak) refocusing*. We prove that these definitions are indeed equivalent. Following a sketch provided by Low, we give a thorough proof of his statement that a strongly causal non-refocusing space-time is homeomorphic to its *sky space*.

A strongly refocusing space-time is refocusing. The converse is unknown. We construct examples of space-times which are refocusing, but not strongly so, at a particular point. These space-times are strongly refocusing at other points. The geometrization conjecture proved by Perelman implies that a globally hyperbolic refocusing space-time of dimension ≤ 4 admits a strongly refocusing Lorentz metric.

We show that the set of points at which a strongly causal space-time is refocusing is closed. We prove that a Lorentz covering space of a strongly causal refocusing space-time is a strongly causal refocusing space-time. This generalizes the result of Chernov and Rudyak for globally hyperbolic space-times.

We compare refocusing and strong refocusing with their Riemannian analogues, $\widetilde{Y}^x$- and Y^x_l-manifolds. A complete connected Riemannian manifold M is called a Y^x_l-manifold if there exist $x \in M$ and $l \in \mathbb{R}^+$ such that all unit speed geodesics starting at x at time 0 return to x at time l. In our work with Chernov and Sadykov we introduce $\widetilde{Y}^x$-manifolds that generalize Y^x_l-manifolds. There we prove that some conclusions of the Bérard-Bergery Theorem for Y^x_l-manifolds hold in fact for $\widetilde{Y}^x$-manifolds. This result is discussed in this thesis.

Following the sketch of Chernov we provide the thorough proof of the statement in their paper with Rudyak that a timelike curve in a globally hyperbolic space-time can be perturbed so that it is transverse to a null-cone and avoids the singular and multiple points of the null-cone. We investigate a possible generalization.