# Construction of the Flag of Nepal in the Conjugate Coordinate System using Mathematica

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## Introduction to Nepal's Flag

Interesting Facts:

- Nepal's Flag is the only non-quadrilateral flag in the world.
- Article 5 of the Constitution of Nepal lays out detailed procedures for the construction of the flag. The instructions are all in the language of geometric proportions. See Appendix 1

Different Shapes within the Flag:

- The flag consists of one inner border and an outer border, one star on the lower triangle, and a crescent moon attached with another star.
- The outer border is colored Blue, the inner border is Crimson, the body of the flag is all Crimson, with the star and the moon colored White.

See Fig (3).

## Introduction to the Conjugate Coordinate System

- A point (X,Y) in the Cartesian Coordinate System is transformed into (x,y) in the Conjugate Coordinate System as:

(1) $$\begin{cases} x = X + iY \\ y = X - iY \end{cases},$$

- Inverse transformation is given by:

(2) $$\begin{cases} X = \frac{1}{2}(x+y) \\ Y = \frac{1}{2i}(x-y) \end{cases}.$$

- Equation of a Straight Line is given by:

(3) $$\tau x + y = \tau r$$

where r = reflex point, is the reflection of origin about the line; $\tau = \bar{r}/r (= e^{-2i\theta})$ = *clinant,* gives the *orientation* of a line in the Conjugate system in a similar way as the *slope* gives the orientation of a line in the Cartesian system. See Fig (1).

- The clinant $\tau$ of a line through points (x1 , y1) and (x2, y2) is given by:

(4) $$\tau = -\frac{y_1 - y_2}{x_1 - x_2}$$

- The following relations describe the relationship between slope *m* of a line in the Cartesian system (*m* real) and the clinant $\tau$ in the Conjugate system ($\tau$ a turn):

(5) $$m = \frac{i(\tau+1)}{\tau-1}, \quad \tau = \frac{m+i}{m-i},$$
$$\tan^{-1} m = \frac{i}{2}\log(-\tau).$$

- The equation of a line with clinant $\tau$ passing through a point (x1 , y1) is given by:

(6) $$\tau x + y = \tau x_1 + y_1.$$

- A straight line can also be represented as ax+by+c=0, where the coefficient of x gives the clinant $\tau$ of the line.
- The square of distance from a point (x1 , y1) to a line ax+by+c = 0 is then given by:

(7) $$d^2 = \frac{(ax_1 + by_1 + c)^2}{4ab}$$

- The equation of a circle with center *k* and radius *r* such that,

$$r^2 = g\bar{g}.$$

where *k* is any complex number and *g* is another complex number other than 0, is given by:

(8) $$(x-k)(y-\bar{k}) = g\bar{g} = r^2$$

which in standard form can be written as:

(9) $$axy + bx + cy + d = 0, a \neq 0$$

See Fig (2).

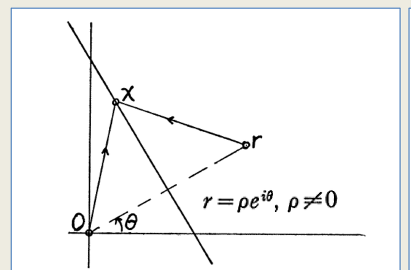

Fig 1: Straight Line in the Conjugate Coordinate System

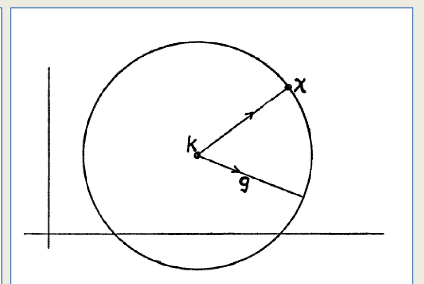

Fig 2: Circle in the Conjugate Coordinate System

## My Approach

I referred to the Constitution as guidelines for the geometric construction, aided by the Construction Sheets at *http://www.fotw.us/flags/np%27.html* (See Appendix 2).

Procedures:

- **Borders**
1. Find all the points that describe the inner border, using translation of complex numbers, and intersections of lines.
2. Transform the inner border as per the direction of the constitution:
   - Translate the points on the inner border edges to points on the outer border edges.
   - Use the translated points to get equations of lines.
   - Intersect required lines to get the vertices of the outer border.
- **Star and Moon**
1. Find all the points that describe the centers and edges of the moon and the star by intersecting lines and circles guided by the constitutional instructions.
2. Use roots of unity, transformed according to constitutional guidelines to get the star shapes.
- **Coloring**
1. Divide the complicated figures into parts and color them separately
2. Bring all the parts together to get the flag

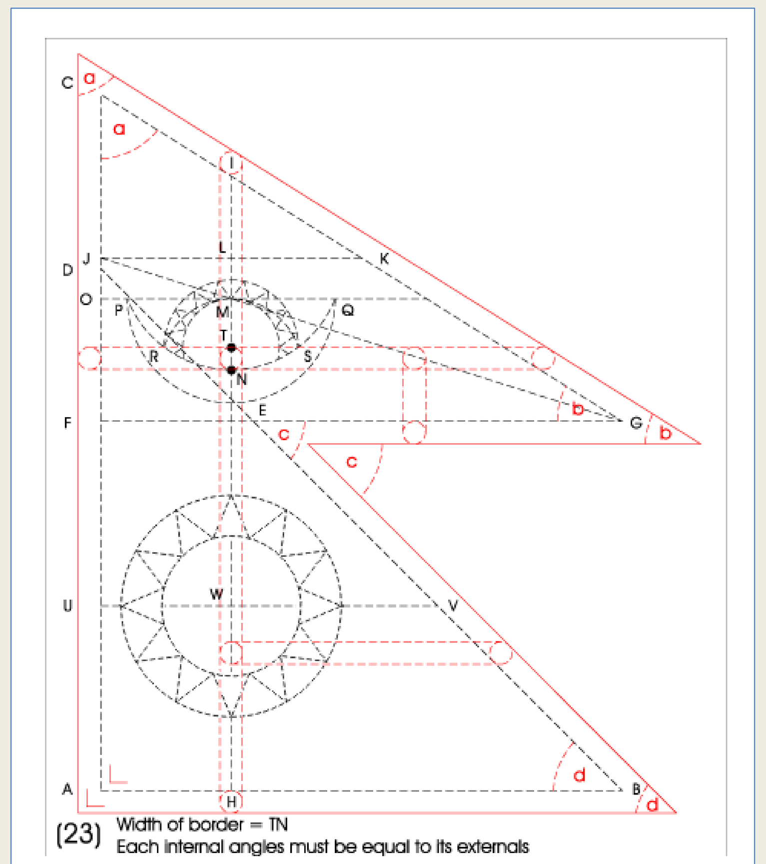


Fig 3: Construction Sheet representing all the points (with lines that describe them) the transformation of the inner border, and the shapes of the moon and the star. [Source: *http://www.fotw.us/flags/np%27.html*]

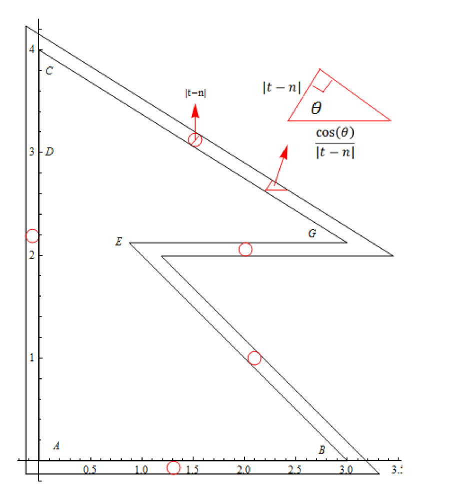

Fig 4: Border Transformation done in Mathematica

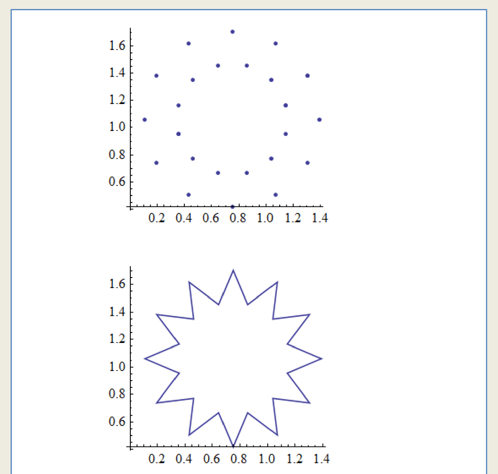

Fig 5: Roots of Unity transformed as required and Joined to give the star.



Fig 7: The lines that join the star portion with the crescent shape.

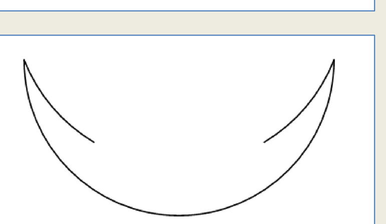

Fig 8: The lines that join the star portion with the crescent shape.

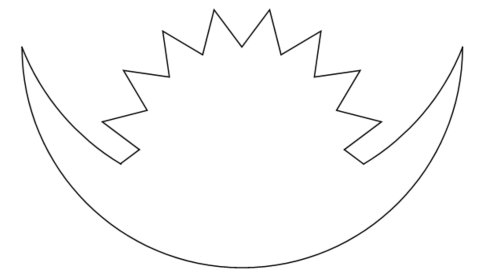

Fig 9: Complete Moon Figure

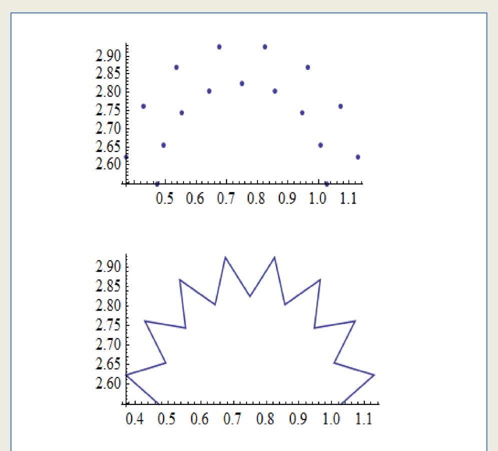

Fig 6: Roots of Unity transformed as required and Joined to give the star of the Moon.

## Explanation of Some of the Steps Involved

### Transformation of the Inner Border to get the Outer Border

- Translation Factor:

A circle with diameter as the length of the Vector T-N (Fig 4)

- Procedures:
1. Translate some points in the inner border to the points in the outer border and then use intersection of lines to get the vertices of the outer border
2. Horizontal translation for the border lines that are slanted was done by first finding the slope of the inner border edges and finding the horizontal translation factor (Fig 4)

### Lower Star and the Star of the Moon

Lower Star:

- Roots of Unity Involved:
1. 12 for the Outer Vertices
2. 24 for the Inner Vertices

- Procedures:
1. Calculate the 12th roots of Unity and transform them to match the required diameter and move the roots to the required center. See Fig (3)
2. Calculate the 24th roots of Unity and transform them accordingly.
3. Find the alternate roots of the 24th roots. The alternate roots are all the odd roots, e.g. 1st, 3rd, 5th,…
4. Join the two lists of roots and Sort them according to the Arguments of the complex roots.
5. Use ListLinePlot in Mathematica to get the required shape.

See Fig (5).

Star of the Moon:

- Roots of Unity Involved and Procedure:
1. 32 for the Outer Vertices.
2. 32 for the Inner Vertices
- Procedures:
1. For inner vertices,
   1. First take the first half of the roots.
   2. Then take the alternate roots. The alternate roots are all the even roots, e.g. 2nd, 4th,…
2. For inner vertices,
   1. First take the first half of the roots.
   2. Then take the alternate roots. The alternate roots are all the odd roots, e.g. 1st, 3rd,…
3. Next, join the two lists. Sort them according to the Arguments of the complex roots.
4. Use ListLinePlot in Mathematica to get the required shape.

See Fig (6).

### Completing the Moon

- Procedures:
1. Use the "hidden" roots of unity in the star portion, i.e. the ones before the start and the after the end of the star figure, to get points for two lines which intersect with a circle, i.e. the upper circle of the crescent shape. The points of intersection are then joined to give the lines that join the star portion with the crescent portion. (Fig 7)
2. Draw the required circles that describe the crescent shape. Lower part is a half circle centered at *M* and the upper part is a partial circle centered at *L.* (Fig 8)

See Fig (9) for the complete shape.

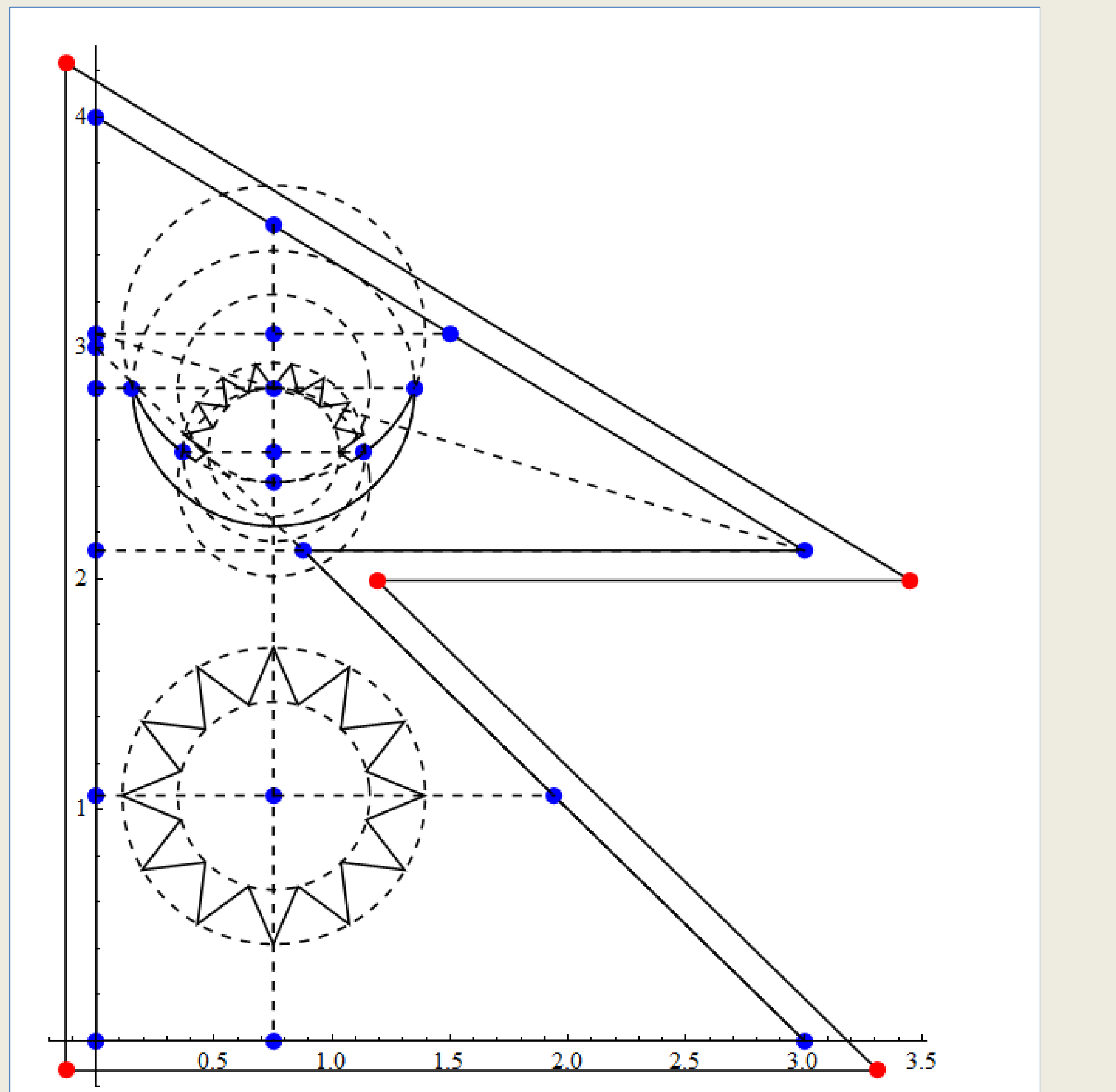

Fig 10: Flag of Nepal showing all the points, lines, and circles that describe the Flag

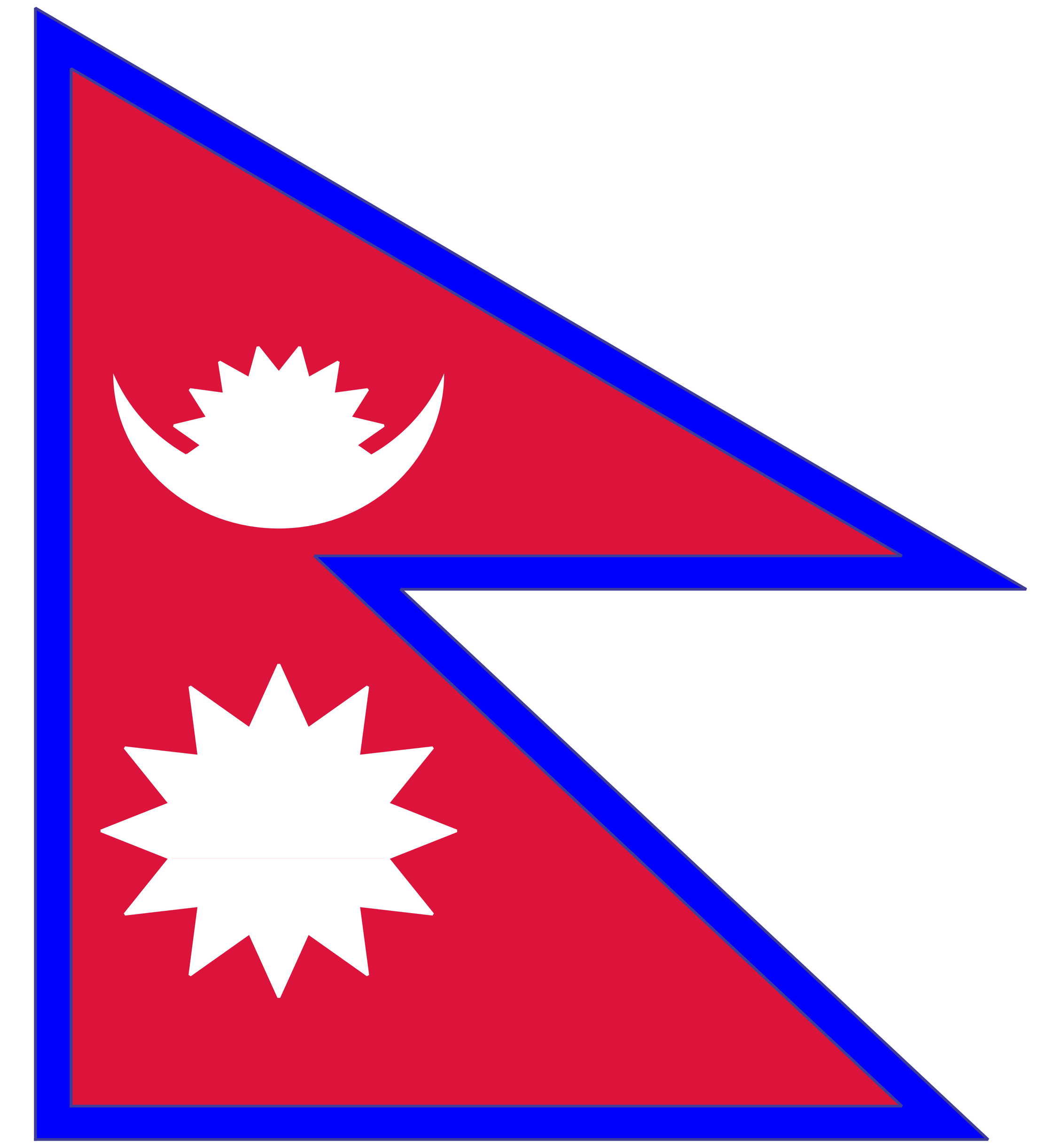

Fig 11: The complete Flag of Nepal drawn in Mathematica after Coloring

## References

*The American Mathematical Monthly* , Vol. 63, No. 9, Part 2: The Conjugate Coordinate System for Plane Euclidean Geometry (Nov., 1956)
Published by: Mathematical Association of America
Article Stable URL: http://www.jstor.org/stable/2309836

Constitutional Guidelines:
http://www.fotw.us/flags/np-law.html

Construction Sheets:
http://www.fotw.us/flags/np%27.html